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The Relationship between English Language and Mathematics Learning for Non-native Speakers

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1. Introduction

In recent years, New Zealand secondary schools and tertiary institutions have enrolled increasing numbers of students for whom English is an additional language (EAL students). There is, therefore, growing interest in the language requirements for successful study and in programmes that will assist these students.

It is a common perception that students from Asian countries, particularly China, enter the New Zealand education system with good backgrounds in mathematics. Anecdotal evidence has suggested that these students take mathematics in New Zealand because they perceive that this subject is less reliant on language skills, and that they have a good background in mathematics in comparison with New Zealand students of the same age.

Another group of EAL students entering the New Zealand education system comes from the Pacific Islands and must adapt to a new culture. Many people have suggested that the language issue is an important factor for these students in their adaptation to New Zealand schools (*see* Appendix 4).

This project was undertaken to better understand the relationship between English language and mathematics learning for both groups of students. We were interested in exploring the extent of any difficulties in learning mathematics attributable to low proficiency in English language, and also discovering particular language features that might cause problems.

Some literature exists that explores this issue at the elementary level (for example, Clarkson, 1991; Setati & Adler, 2001), but there is little work at the senior secondary or tertiary levels. In New Zealand, many of the EAL students arrive in our education system in the final years of secondary school or directly into tertiary institutions.

The senior researchers in this project had already undertaken some preliminary research into learning mathematics in English at first year undergraduate level (Barton & Neville-Barton, 2003). The Teaching & Learning Research Initiative (TLRI) has provided an opportunity to extend this research, and to involve teachers from a variety of environments. Researcher and teaching practitioner partnerships were established to encourage teachers to develop as critical professionals reflecting on their practice, in particular, with respect to language issues in their classrooms.

2. Aims and Objectives

The research theme introduced above relates directly to the TLRI principles. Principle One concerns strategic value to education in New Zealand. The research responds to the increasing diversity of students in the New Zealand education system, and attempts to further understand how linguistic diversity affects their learning. The project addresses the inequalities of this situation.

Principles Two, Three, and Four relate to the quality of the research. By building on work of experienced researchers in the team, and investigating a variety of learning situations, this study was designed to reach substantive findings which will directly affect future practice.

Principles Five and Six relate to the role of teachers as researchers. This project needed to promote significant development of all its teachers as researchers.

Therefore, the aims of this research project were:

- to examine the impact and nature of language factors in the learning of mathematics for EAL students;
- to produce recommendations for mathematics teachers of EAL students;
- to produce guidelines for the design of language support programmes; and
- to develop a group of teachers with an interest in language and mathematics and the skills to continue researching this issue.

Research Questions

1. It is known that there are disadvantages for EAL students learning in classrooms where English is the language of instruction. Elder (1993) and Graham (1987) estimate the variability in academic performance due to English language ability is up to 10 percent for university students, and that it is higher for humanities and social science subjects in comparison with mathematics or science subjects. However, Barton and Neville-Barton (2003) suggest that the disadvantage due to language may be just as high in mathematics as in other subjects. Therefore the first research question is as follows.

   What is the extent of the disadvantage in mathematics learning due to low English language proficiency at the senior secondary and university undergraduate levels?

2. There is a considerable literature on the linguistic features of mathematical discourse in English (e.g., Clarkson, 1991; Dale & Cuevas, 1987; Halliday, 1975; MacGregor & Moore, 1991). There is also a literature that examines discourse features in different languages (e.g., Galligan, 2001; Setati & Adler, 2001). However, while features have been described, there has been limited research on the difficulties these cause for mathematics learners, particularly at senior levels. Abedi (2001), for example, is one of a limited corpus, and has a focus on elementary mathematics. Naudé (2003) is one of a group that compares EAL students with native speakers of English (L1 students). However, the different language features affecting students from different foreign language backgrounds has not been directly addressed.

   What specific language features cause difficulty for particular groups of senior and undergraduate EAL students learning mathematics, and how do these compare with language difficulties experienced by L1 students?

3. Another aspect of this study was to investigate whether it is general language ability or specific technical language ability that is most important in learning mathematics in another language. The tests used for EAL students entering English-speaking educational institutions (for example, the Cambridge International English Language Testing System (IELTS)) are measures of general academic proficiency. The literature on specific mathematical discourse has been referred to above. These two linguistic considerations generated a third question.

   What is the relative importance of technical language knowledge compared with general language proficiency in the learning of mathematics at senior secondary and undergraduate levels?

4. Earlier research at undergraduate level by Barton and Neville-Barton (2003) indicated that students were unaware of the extent of their disadvantage due to low English proficiency. It is possible that this is a result of a belief that mathematics learning is language free. Such unawareness is a severe limitation to overcoming any language disadvantage, so it is important to know whether it is widespread.

What awareness do secondary and undergraduate EAL students have of the difficulties they face due to low English proficiency?

5. Mathematics classes in urban New Zealand schools and all tertiary institutions have significant numbers of EAL students. For example, in Barton and Neville-Barton's (2003) research at first-year undergraduate level at the University of Auckland, over 60 percent of their large random sample were EAL students. Understanding the issues faced by these students is critical for effective teaching and should translate into the provision of support.

   What practical steps can be taken by teachers in mathematical learning environments to ameliorate language effects; and what support services can be provided by educational institutions?

3. Research Design

The Research Team

The study involved an initial research design team of five practitioners with research experience, each from a different institution and a research co-ordinator from a sixth institution. The design team was identified by Pip Neville-Barton and Bill Barton as successful, active mathematics teachers who had had some contact with university post-graduate programmes. Each member of the team co-opted other teachers from their institutions who had an interest in the topic and who wanted to develop research skills. Each group undertook an independent study in their institution, although the topics were closely related under the project aims, and some research instruments were shared.

Seven meetings were held during the course of this project. The research design team of six met initially in December 2003 and again in February 2004 to discuss the research issues, to distribute and discuss the literature, and to draft a timeline and a research plan that would take into account the different ethnographic features of each institution. Before the full team meeting in March, one of the original research design team members had to withdraw from the project. However, at this stage, Elaine Vine from Victoria University expressed an interest in joining the project with teachers from Wellington Girls' College. This was welcomed.

The total team therefore, comprised 16 people as listed below (initial design team is underlined):

Unitec, New Zealand:	Pip Neville-Barton (applied linguist and project co-ordinator
University of Auckland:	Bill Barton, Bob Chan, Chris King, Viliami Latu (mathematics lecturers)
Macleans College:	Mark Phillips, Bruce Dixon, Vaughan Mitchell (mathematics teachers)
Auckland Girls' Grammar School:	Jushi Hu, Anne Blundell (mathematics teachers)
Tangaroa College:	Sosefina Paletaoga, Rasela Lafaele, Havili Ofamo'oni (mathematics teachers)
Victoria University:	Elaine Vine (applied linguist), working with

Wellington Girls College:	Monica Luxford (mathematics teacher), Marianne Devere (ESOL teacher)

There were five further full team meetings where selected literature was discussed, research instruments were developed, findings were shared and discussed, and writing up of individual projects began.

Ethics

Ethics approval was granted through the Unitec Research Ethics Committee on 14 February 2004. Information sheets and consent forms were sent to the principals of the schools, to the HOD Mathematics at the University of Auckland, and Head of School of Linguistics and Applied Language Studies at Victoria University of Wellington. Information sheets and consent forms were also distributed to the student participants before the data were gathered.

Research Participants

The study focused on senior secondary students and third year undergraduate levels of mathematics learning. Participants were mainly from Mandarin, Tongan, or Samoan language backgrounds although two studies included English native speakers. (*See* Individual Study summaries below.)

Methodological Approaches

Two methodological approaches were applied in the project—the investigative approach and the scientific approach.

The investigative approach was used to find out more about the situations of different EAL students learning mathematics. It involved looking closely at individuals' experiences of mathematics learning in English, observing their classroom situations and looking at their work, asking them questions, and talking to them about their successes and difficulties.

The scientific approach was used to test particular hypotheses that had been developed in earlier research or in the literature. In particular the extent of any disadvantage for EAL students compared with L1 students, needed to be verified in different ways. Also, tests were designed to show whether in fact particular pre-identified features of language actually did cause difficulties amongst particular populations. This involved administration to large enough groups so that statistically significant results and generalisations could be made.

The original research plan was adapted to accommodate the particular interests and skills of the researchers and their student population. Therefore, although the aims of the research were the same for all five studies, the research tools were individualised for each institution.

Although all the studies had both scientific and investigative features, three of the studies were primarily investigative. One examined an area of mathematics and mathematical discourse not previously studied (proof and argumentation in third year undergraduate mathematics), a second focused on Paskifika-speaking students, using the personal experience of the Pasifika researchers, while a third made use of the applied linguistics background of the researchers to conduct in-depth interviews with Mandarin-speaking students. Two studies were primarily scientific, one using mathematically matched Mandarin-speaking and English L1 groups to discover the specific features of mathematical discourse that resulted in statistically significant differences, and the other using a bilingual research design to look closely at English-Mandarin discourse differences.

Observations, tests, questionnaires, and selected interviews were all used as data collection techniques.

Individual Study Summaries

Below are brief summaries of the studies conducted in each institution. Fuller reports are attached as appendices. Further information can be accessed on request.

Auckland Girls' Grammar School

This study involved 40 Years 12 and 13 Chinese Mandarin-speaking students. The project was fortunate to have a bilingual native Mandarin-speaking teacher, Jushi Hu, who was able to write parallel tests in Mandarin and English. These tests were administered in two sittings seven weeks apart. At each sitting half the students did the English and half the Mandarin version, swapping over in the second test. The analysis focused on comparing students' performance on the Mandarin and English versions of the test. Group interviews were conducted to gather further insight into the test responses.

The study indicated that these students experienced, on average, a 15 percent disadvantage in overall performance in the English test compared to their performance in the Mandarin test. The syntax of mathematical discourse appeared to cause more problems than vocabulary. The teachers were surprised by some of the misunderstandings revealed in the interviews. There was also lower overall performance, indicating that this group of students is not as mathematically competent as expected by their teachers. Interviews revealed that some students had not had the higher level background usually associated with students from China.

Wellington Girls' College

This study involved 13 Years 12 and 13 Chinese Mandarin-speaking students. The test and administration paralleled that at Auckland Girls' Grammar School, with a shorter time between tests. It included a self-reporting of students' understanding of mathematical instruction. Twelve of the students were interviewed individually and in depth. The analysis focused on the nature of their language difficulties and the strategies the students thought would help their learning.

This study confirmed the disadvantage for students when doing mathematics in English, with a difference of 12 percent on this smaller sample. The interviews revealed particular misunderstandings, a narrow understanding of some concepts, and many strategies for coping with their lack of comprehension. The students self-reported only a little difficulty on average in understanding mathematics in English, despite the problems revealed in their test performance and interviews.

Macleans College

This study involved testing 135 Year 13 students from a variety of language backgrounds. The test collected demographic information and Year 12 grades, and tested mathematical syntax and vocabulary, contextual problems, and problems with redundant information. The analysis was restricted to a Chinese group (14 students) and an English group (17 students) with parallel mathematical ability based on Year 12 grades.

Only three of 32 items showed a significant difference at the 1 percent level—on all three items the English group outperformed the Chinese group. On a further six items there was a significant difference at the 5 percent level; two of these were done better by the Chinese group. No other items showed significant difference. An examination of the individual items revealed the main problems for Chinese students were: prepositions, word order, and interpreting context.

Tangaroa College

This study involved observations of two Year 12 mathematics classes, the administration of two questionnaires to the 42 Pasifika students in these classes, and interviews with 16 students. Initial observations and researcher experience led to a hypothesis that vocabulary was the most important issue for these students. The first questionnaire tested this feature, mathematical syntax, and mathematical word problems. The second questionnaire tested specific discourse features in word problems. Students received the questionnaire in English with a translation into their first language.

The study indicated that vocabulary on its own, particularly instructional vocabulary, was not as problematic as anticipated. Rather it was the combination of syntax and technical vocabulary that caused difficulties. Word problems involving implication were the hardest for the students to solve. During the interviews it emerged that low general proficiency in both languages could also be a significant factor in learning mathematics.

The University of Auckland

This study involved observation of two third-year university mathematics courses. Twelve Chinese-speaking students from one course were then asked to self-report their understanding of the course and were tested on specific mathematical items. English language proficiency results were also available for these students. Follow-up testing of two large courses was undertaken in the second semester to confirm the results and to enable a comparison with English L1 students to be made.

Significant differences were found in third-year classes compared with first-year ones, in that mathematical understanding was much more deeply embedded in the language of the lecturer and texts. The result of the initial testing showed that the disadvantage experienced by the EAL students due to language is higher than expected, and was severe for those students with lower English proficiency. All students appeared unaware of their difficulties. The follow-up testing confirmed these results and indicated that the L1 students did not have any language problems.

4. Findings and Limitations

All five studies offered quantitative or qualitative evidence that EAL students suffer a disadvantage in mathematics learning due to language difficulties. The extent of this disadvantage was measured as 12 percent and 15 percent in two of the studies, and this corroborates with earlier work by Barton and Neville-Barton at first year university level. The benefit of the complete TLRI project is that the disadvantage was investigated in several different ways: a bilingual test, mathematically matched English and Chinese groups, mathematical syntax and logic tests, and interview data (Research Question 1).

Four of the five studies sought evidence of the EAL students' perceptions of their own understanding of English mathematical discourse. All evidence indicated that they do not realise the extent of their difficulties. We suggest that raising their awareness of this issue is a prerequisite for improving the situation (Research Question 4).

All studies reported that students in general performed worse than the teachers/lecturers anticipated. There was evidence from interviews that, contrary to assumptions, some students did not have the background required for senior levels of mathematics.

Three of the studies revealed that both general and technical English were factors in the problems experienced by EAL students. The Macleans College study, with students of uniformly higher mathematical proficiency, indicated that general English was a bigger factor. The Auckland Girls' Grammar School study, with students of varied mathematical proficiency, reported that the mathematical discourse was a bigger factor. The indication is that the type of language causing difficulty is related to the mathematical proficiency of the student (Research Question 3).

What specific language features cause difficulty? (Research Question 2). The features varied across the studies, and appear to depend on the mathematical level as well as the home language and English language proficiency levels. Each study reports particular items—*see* appendices. Vocabulary on its own is not the big issue that was anticipated. However it was a component of the difficulty experienced with understanding mathematical discourse as a whole.

As suggested by the literature, prepositions and word order were key features causing problems at all levels. So also were logical structures such as implication, conditionals, and negation, both at senior secondary and third year university levels. Mathematics couched in everyday contexts caused the expected problems.

The three studies with Chinese-speaking secondary students all reported anomalies in the test item using the word "gradient". This question was the only one answered significantly better when presented in English rather than in Mandarin. It was suggested that this was because this concept was not taught in China, but was a feature of the mathematics courses taken in English. Further research needs to be done to investigate the conditions under which concepts taught in one language transfer to another language.

Some interview data, along with the experiences of the teacher/researchers, indicate that students having difficulty with language "switch off" in class, relying on texts or handouts. They tend to focus on procedures and approach mathematical problems in tests by trying to recognise a suitable procedure without trying to understand the context. For example, the word "less" may produce a response of "subtract" when this is inappropriate. Language difficulties also seem to limit students' mathematical solving techniques; for example, such students have difficulty drawing a diagram and are restricted to symbolic mode.

Limitations

Although the separate studies in this research are consistent in their broad conclusions, for example about self-reporting little difficulty with English in mathematics while actually experiencing significant problems with the syntactical aspects of mathematical discourse, each study individually has too small a sample to draw broad generalisations.

The levels of disadvantage were tested in different ways; for example, by comparing EAL and L1 matched groups, and by comparing Chinese-speaking students' ability when the tests were presented in English and in Mandarin. Furthermore, the levels of disadvantage evidenced in the various studies are also consistent (between 10–20 percent). However, the tests used have not been comprehensive. That is, they have not covered the full range of English language presentations of mathematical discourse (for example, no measure of understanding teacher talk was attempted), and also, not all aspects of mathematical content were covered (for example, statistics was not a part of any of the studies).

It should also be noted that this study took place in Auckland at a particular time. It is likely that there is a particular type of EAL student in the study. Thus, for example, the Chinese-speaking students who participated cannot be assumed to be representative of all Chinese-speaking EAL students learning mathematics in English.

The studies were conducted by practising teachers who have a little research experience, but who are not full-time researchers. There are acknowledged shortcomings of research methods.

Despite these limitations, the studies are, we repeat, broadly consistent, and they also confirm previous research at a similar level. We are confident, therefore, in making the recommendations below.

Recommendations & Further Research

Researchers in each of the studies developed their own set of recommendations and suggestions for further research. They are not repeated here—*see* appendices. However, there are some ideas that are present throughout this project, and which are important enough to be made into general statements with strong support.

There is no doubt in the minds of the research team that this has been a productive study in personal terms (see Capacity Building below). The mode of involving practising teachers in research about aspects of their particular situation with the active involvement of more experienced researchers and administrative backup was very successful.

1. *Teachers undertaking supported research, specific to their classroom and subject, is an effective mode of professional development, and should continue to be a significant part of any development programme.*

The several studies in this project provide strong evidence to back up other research that EAL students learning mathematics in English suffer considerable disadvantages that are not recognised, either by the students or by their teachers. The myth that mathematics is language free can no longer be sustained. EAL students need support in this area, as in others.

2. *Resources need to be allocated to supporting EAL students in this area (from Pasifika, Chinese, and many other language groups).*

   Specific aspects of this support indicated by this research are as follows.

   2.1 *Better understanding of these students' language and mathematics proficiency at the time they enter New Zealand classrooms—and hence better placement of these students.*

   2.2 *The development of special courses in English mathematical discourse, with particular focus on making links between mathematical discourse in the students' home language and in English.*

   2.3 *The development of in-service programmes for teachers to increase their awareness, and to give them strategies, to support EAL students in their classroom.*

This project has uncovered "an iceberg" in the words of one of the teachers/researchers. The issue of language in mathematics classrooms has been acknowledged in the past, but the extent of the problem and the specific nature of the difficulties have not been properly investigated. The various studies in this project show clearly that the issue is both complex and situation-dependent. We need to know a lot more about the issues before we can deal with them properly.

3. *Further research in this area is warranted.*

   3.1 *Further research is needed into the mathematical discourse of Pasifika languages.*

3.2 *Further research is needed into the relationships between Mandarin and Cantonese mathematical discourse and that in English, and whether students' difficulties arise from these relationships.*

3.3 *The effectiveness of courses for students designed to support their learning of mathematics in English needs to be properly evaluated.*

3.4 *The feasibility and effectiveness of providing opportunities for students to discuss mathematics in their home language as part of the pathway to learning mathematics in English, needs to be investigated.*

These recommendations address the principle of reducing inequalities in education in New Zealand. This is a theme of strategic importance in education in New Zealand emerging from the diversity of our student population.

The recommendations also focus squarely on the teacher. It is only through their increased interest in this issue, and their awareness and understanding of the problems of language in mathematics learning that progress will be made. This project was an example of effective collaboration between researchers and practitioners, in which each was required to communicate clearly their experience, and acknowledge the other's point of view. These aspects reflect Principles 5 and 6 of the TLRI Priorities.

Capacity Building

As a team, we explicitly discussed the benefits of this research on more than one occasion in our meetings. In addition, towards the end of the project, we gathered written feedback using a questionnaire entitled "Impact of Research on Teaching".

We were left with no doubt that this has been a positive involvement for the teacher/researchers, and compares extremely favourably with other professional development experiences. These comments apply to all the teachers in the project, not just to a majority opinion. Part of the evidence for this is that all teacher/researchers worked far in excess of the "paid" time allocation for this project. Furthermore, the researchers who are not teachers were left with a significantly enhanced appreciation of the realities of EAL students in mathematics classrooms.

There were four distinct areas in which the project made a difference: teachers becoming better practitioners; teachers being stimulated to work together and to find out more about language issues; teachers wanting to undertake further formal study; and teachers becoming better researchers. What follows is a selection of the evidence, much of it in the teacher/researchers' own words.

Better Practitioners

> I now try to speak slowly and pronounce words clearly.

> I write meanings on the board. Make those meanings clear, repetition of these key words is vital. Encourage mathematical discourse amongst the students.
>
> I have seen [the students] more problems more clearly and in detail since I have been doing this project. I pay more attention to words or syntax used in maths problems.
>
> It has made me think very carefully about the instructions I give to my students verbally or on the board.
>
> I am more appreciative of the gaps in their mathematical language, and need to find ways to encourage them to ask for help, or tell me when they do not understand a term used.

Interest and Collaboration

> I found sharing extremely interesting.
>
> I feel my job description has expanded to a teacher of English as well.
>
> A big plus has been and continues to be the opportunity to work with colleagues across sectors and across institutions.
>
> I always look forward to [a co-researcher] coming to school and…talk about how the students performed.
>
> Interesting and stimulating. We have scratched the surface and need more.
>
> I want more data.

Further Formal Study

> It took me to the level where I think that learning more and having to research more into this issue is very beneficial. It even leads me to pursue further studies into this field.
>
> It has inspired me to start a PGDipSci with aspirations to complete an MSc in maths ed.
>
> Yes, [in the future I want to do] more research to complete my Masters.

Five of the team will be active in postgraduate study in 2005. Three of these are undertaking further research related to their work on this project.

Teachers Developing as Researchers

> It's been really good for me in that it has allowed me to work with other more experienced researchers, but still have an important role to fulfil. It's given me the opportunity to participate in a supportive environment, and the luxury of having other perspectives to discuss and consider. Also, because the research is bigger than just me, it gives the whole endeavour an extra bit of legitimation and importance.
>
> I feel that we've found an iceberg, and now something needs to be done.
>
> Involvement in research has been good for changing practice because there is a focus on my subject. Generic professional development material is limited in its usefulness to me.

This process was longer than much professional development, and grappled with something in depth.

It has been good to be able to ask the students questions about what they are doing and how. Doing it as research means that there is a structure and a concerted effort to come up with thoughtful questions/answers.

5. References

Abedi, J. (2001). The language factor in mathematics tests, *Applied Measurement in Eduction, 14*(3), 219–335.

Barton, B. & Neville-Barton, P. (2003). Language issues in undergraduate mathematics: A report of two studies. *New Zealand Journal of Mathematics, Vol 32*, Supplementary Issue, 19–28.

Barton, B., Chan, R., King, C., & Neville-Barton, P. (2004). The mathematical discourse of advanced undergraduate mathematics. In I. Putt (Ed.) *Proceedings of 27th Mathematics Education Research Group Conference*, (pp.79–86), Townsville: James Cook University.

Clarkson, P. (1991). *Bilingualism in mathematics learning*. Geelong: Deakin University Press.

Dale, T., & Cuevas, G. (1987). Integrating language and mathematics learning. In J. Crandall (Ed.), *ESL through content-area instruction* (pp. 9–52). Englewood Cliffs, NJ: Prentice Hall Regents.

Elder, C (1993). Language proficiency as a predictor of performance in teacher education. *Melbourne Papers in Language Testing, 2*(1), 68–89.

Galligan, L. (2001). Possible effects of English-Chinese language differences on the processing of mathematical text: A review. *Mathematics Education Research Journal, 13*(2), 12–132.

Graham, J. (1987). English language proficiency and the prediction of academic success. *TESOL Quarterly, 21*(3), 505–521.

Halliday, M. (1975). Some aspects of sociolinguistics. In E. Jacobsen (Ed.), *Interactions between language and mathematical education: UNESCO Report No. ED-74/CONF-808*, 64–73, Paris: UNESCO. Reprinted 1978 as Sociolinguistic Aspects of Mathematical Education, in M. Halliday, *Language as social semiotic*, (pp. 194–204). London: Edward Arnold.

MacGregor, M., & Moore, R. (Eds.) (1991). *Teaching mathematics in the multicultural classroom: A resource for teachers and teacher educators*. Melbourne: Institute of Education, The University of Melbourne.

Naudé, A. (2003). The influence of second language mathematics teaching on calculus performance. *Remarkable Delta: 03 Communications* 199–204. Auckland: International Delta Steering Committee.

Setati, M., & Adler, J. (2001). Between languages and discourses: Language practices in primary multilingual mathematics classrooms in South Africa. *Educational Studies in Mathematics, 43*(3), 243–269.

Appendix 1: Auckland Girls' Grammar School

Comparing Mathematics Performance in Two Languages

Jushi Hu and Anne Blundell

Department of Mathematics, Auckland Girls' Grammar School

Abstract

This study examines the language factors in senior secondary mathematics for students who do not have English as their first language. It involved 40 Years 12 and 13 Chinese students at Auckland Girls' Grammar School. Parallel tests in English and Chinese were given to the students some weeks apart. The study indicates that these students experienced, on average, a 15 percent disadvantage in overall performance in the English test compared to the Chinese test. Some specific language features causing difficulty emerged from the analysis.

Background

Auckland Girls' Grammar School is a decile 4 state school situated within a 2 kilometre radius of the Auckland CBD. Our focus was the difficulties our Chinese Mandarin speakers encounter when learning mathematics in English.

The girls all spoke Mandarin as a first language. Their length of time living and studying in New Zealand varied between less than 1 week and up to 5 years. A large majority of these Mandarin speakers had already studied up to Year 11 mathematics in China. There was a big variation in the students' mathematical abilities in their first language. We believe their abilities could be modelled by a normal distribution. This needs to be considered, as a common perception is that "Asian" students are typically well drilled in mathematics problems. The students' life experience is also a factor when considering their responses and ability to solve mathematical problems based in a New Zealand English context.

The teacher/researchers, Jushi Hu and Anne Blundell, both qualified as secondary mathematics teachers in 2002. Jushi is a native Mandarin speaker with 12 years' experience lecturing mathematics in China. She has been teaching for 4 years at Auckland Girls' Grammar School. Anne is a native New Zealand English speaker with 3 years' teaching experience in New Zealand and 1 year in England. She has been teaching at Auckland Girls' Grammar School since February 2004.

When considering students' difficulties with mathematics in English, Jushi's thoughts, prior to the research, were that every word, phrase, or the syntax encountered posed potential room for error in understanding the question, and therefore reduced students' chances of solving problems correctly. She also thought students lacked confidence when attempting to understand English language problems and experienced uncertainty when beginning to solve these problems.

Anne's major concern was that the language used in texts and assessments posed problems for native English speakers and therefore was even more difficult for Chinese students. Anne was aware that extra tuition and explanation of simple everyday terminology in mathematics greatly improved understanding for second language students. She found spending enough time with them difficult in a classroom environment, but essential for their progress and advancement in this subject.

The Study

A 45-minute assessment was designed in which 14 items were chosen from standard issue textbooks. They were modified to try to identify the English language aspects that the teachers believed the students would have difficulty understanding. In particular it was intended to investigate Mandarin speakers' ability to understand English vocabulary and syntax. Seven questions looked at straightforward vocabulary understanding. The remainder required an understanding of syntax from a simple to an advanced level. A mixture of English and mathematical skills was required in order to answer the questions correctly.

Two versions of the test were produced: an English version and a directly translated Mandarin version. Forty Mandarin-speaking students from Years 12 and 13 volunteered to participate. Students sat both tests separated by a 7-week interval. Half sat the English version first; half sat the Mandarin version first.

This was followed by interviews conducted to gain further insight into what features of the questions the students had difficulties with or did not understand.

Results

Table 1 indicates that there were a lot of items that had no response in both versions. Six items out of fourteen resulted in significantly better correct responses using the Chinese version of the test. Overall, there was a difference of performance between English and Mandarin of 14.8 percent, with students performing better in the Mandarin version.

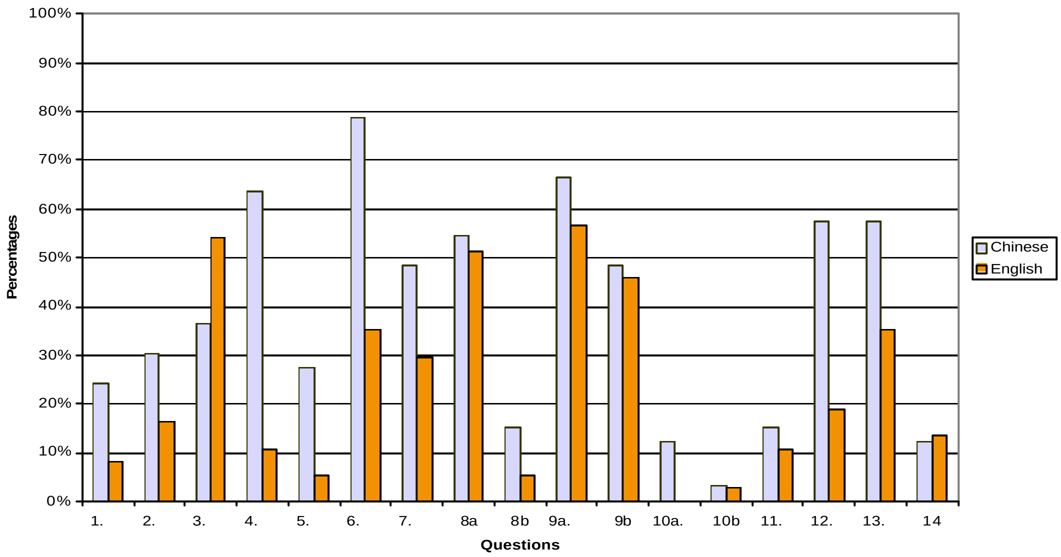
Graph 1 (below Table 1) shows that six questions out of 14 were answered correctly by 50 percent of candidates in the Chinese version. Only three questions out of the 14 were answered correctly by 50 percent of candidates in the English version.

Table 1 **Responses for each question in the test**

Questions	Correct responses		Differences
	English	Chinese	
1	8%	24%	16%
2	16%	30%	14%
3	54%	36%	-18%
4	11%	64%	53%
5	5%	27%	22%
6	35%	79%	44%
7	30%	48%	18%
8a	51%	55%	4%
8b	5%	15%	10%
9a	57%	67%	10%
9b	46%	48%	2%
10a	0%	12%	12%
10b	3%	3%	0%
11	11%	15%	4%
12	19%	58%	39%
13	35%	58%	23%
14	14%	12%	-2%

(Negative values in the Differences column indicate better performance in English.)

Graph 1 **Comparison of Correct Responses**



To investigate the difficulty in understanding and solving mathematical problems, the test items were sorted into two categories: those in which vocabulary was identified as likely to cause problems, and those with complex syntax. Item 3, although a vocabulary item, was not included in that category and is discussed separately below.

Table 2 shows how students performed in the items featuring mathematical vocabulary. The results show an average 13.7 percent difference in performance between Chinese and English.

Table 2 **Responses on vocabulary items**

Vocabulary focused items			
Questions	**Correct responses**		**Differences**
	English	Chinese	
1	8%	24%	16%
2	16%	30%	14%
5	5%	27%	22%
7	30%	48%	18%
10a	0%	12%	12%
10b	3%	3%	0%

Table 3 indicates a greater disadvantage (on average 25.3 percent) that Chinese students experienced in answering questions involving syntactical structures in the English test compared with the Mandarin version.

Table 3 **Responses on syntactical items**

Syntactically focused items			
Questions	**Correct responses**		**Differences**
	English	Chinese	
4	11%	64%	53%
6	35%	79%	44%
8a	51%	55%	4%
8b	5%	15%	10%
11	11%	15%	4%
12	19%	58%	39%
13	35%	58%	23%

Discussion

Overall Differences

There was an overall difference in performance of 14.8 percent between the Mandarin and the English test performances, with the better performance in Mandarin. This confirms the Barton and Neville-Barton studies of first year university students that language affects mathematics as much as it affects other subjects which are more obviously language rich.

Vocabulary and Syntax

In this test, students performed on average 13.7 percent better on vocabulary-focused items in the Mandarin version. In the items with more complex syntax the difference averaged 25.3 percent. This was confirmed in the interviews, where students indicated that they had most difficulty with understanding the contextual questions with combinations of complex phrases, syntax, and technical mathematical vocabulary.

The questions contained a wide range of technical vocabulary. Some students had never encountered these words in their learning experience of English. Also, these mathematical terms often have other general English meanings. For example, questions 10a and b use the words "depression" and "elevation" which have multiple meanings and which had not been taught to these students.

Questions 5 and 7 tested students' understanding of the words "coefficient", "perimeter," and "diagonal". These were answered significantly better in the Chinese version of the test. This is surprising as these words are basic terms which are in frequent use at this level of study.

Mathematical questions in English have a strong tendency to use complex syntactical structures. From Jussi's experience, Chinese syntactical structures are simpler and more regular for mathematical questions than English. An example is in question 4 where the sentence in English reads "The square root of one half of a number is 8." This caused the most difficulty for students (64 percent of students answered correctly in Chinese and only 11 percent answered correctly in English). Similar examples are found in questions 6, 8, 11, 12, 13. In particular, the sentence "six times as much milk as syrup" in question 6 is extremely hard to understand from the view point of Chinese grammar. In Chinese, this would be written, as "milk is syrup six times." The large difference (44 percent) in answering this question correctly, is likely to be because of the complexity of the proportional relationship of the question.

However, it was surprising to find, through the interviews, that some questions we believed would pose problems because of the syntax were difficult for the students because of the vocabulary. Students did not understand what "square root" meant. When translated into Chinese or given in symbols, they answered very well. Another feature of note occurs in question 12, where a deep understanding of the mathematical context and complexity of the sentence is required to fully grasp the question. Consider the sentence “An isosceles triangle has a base that is three-quarters the length of one of the equal sides.” It has combined technical vocabulary, complex syntax, and prepositions. Students need to understand the nature of an isosceles triangle, that is, that two sides are equal. Also, “three-quarters the length of one of” is a hard syntactical structure for students to comprehend due to the nested structure of the question.

Students also said in the interviews that they tended not to attempt the question at all if they felt they encountered a key mathematical word that they did not recognise. Students focused on the specific word and because there was not enough context or any visual aids it did not allow even a “best guess” or attempt at the result. Diagrams would have helped them further understand what is required.

Most students had not learnt any mathematical terms in English in their experience of learning English in China. Some students did not understand the meanings of basic words at junior level in mathematics, for example, fraction, decimal, division, and triangle. In subsequent discussion with students about their in-class worksheets Anne was often surprised at their lack of knowledge of words such as “parallel” and “perpendicualar” and felt that this explains why many students do less well in coordinate geometry compared with algebra.

The above illustrates the complexity of analysing in detail the role of language in mathematics. There is a complex interaction between language features, context features, mathematical knowledge, and use of symbols.

Poor Mathematical Performance

We assume that students’ ability in mathematics is also essential for total comprehension. Poor mathematical ability for some students hindered their performance. The test results in both languages gave a lower than expected level of correct responses for students at this level.

The students’ background may have contributed to the results of this study. There is a large variation in their mathematical abilities. In order for them to study in New Zealand their mathematics was possibly abandoned to further their English studies. Also the English proficiency of the new arrivals is poor in general. Several of the Year 12 students had been here for only 2 weeks. When arriving in New Zealand they are confused and lack the ability to access and follow the new curriculum in classes. It takes them a period of time to adapt to a new learning environment and teaching style.

In China, all schools and classes are streamed. These students come from different academic backgrounds. A minority of Chinese students come from accelerated classes and have strong abilities in mathematics. The majority of students are at an average level in terms of mathematical ability and all students have sound arithmetic skills developed at primary school level. However, we should be aware that some students may have extremely low ability in mathematics; from the interview we found some students had only learnt mathematics up to primary school level or Year 9 or Year 10 in China.

Chinese students entering New Zealand classes are assumed to be competent at mathematics. Their silence in class is taken to mean that they are not confident in general oral English, or a function of the teacher-centred style used in China. This research suggests that part of the reason may also be that they do not understand the mathematics.

Question 3 and the word "gradient"

Question 3 was the one question in which students performed significantly better in English. The evidence is that this is due to the particular word "gradient". Many of the students had been studying the concept of gradient at Years 12 and 13 in English and it is a major conceptual component of any senior maths course. The interview suggested students had learnt the concept of gradient at Year 11 in China, but had not recalled the Chinese meaning. Note from the above that several students had not reached Year 11 in China. Most Year 13 students could answer this correctly.

Summary

This study is undertaken at senior levels at secondary school. The study shows that these students suffer about a 15 percent disadvantage when being tested in the English language. There are important immediate challenges to secondary schools with Chinese students. The study has revealed that mathematics is not simply a symbolic language and that particular vocabulary, syntax, and discourse present challenges for Chinese learners.

The following recommendations are made based on the researchers' experience.

Recommendations

- Students should be assessed on arrival for both their English and mathematical knowledge and placed into appropriate classes. (It is noted that separate assessments for these two abilities may need to involve mathematics assessment in Mandarin.)
- Group and class participation and fostering an enquiring culture within the classroom should be encouraged. For example, students might keep a question log in the back of their books and present their questions on a regular basis.
- The Auckland Girls' Grammar School half-year foundation course in Level 12 mathematics should be continued, including the employment of a teacher aide who speaks Mandarin.
- A glossary of basic technical mathematical terms in English and Chinese, preferably with visual aids, should be compiled and handed out at the start of the course. This could be supplemented by the teacher writing on the board relevant items at the beginning of each session.
- All teachers need ongoing professional development with respect to the teaching of EAL students.

Appendix 2: Wellington Girls' College

Investigating Mathematics Performance in Two Languages: Student Voices

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Victoria University Wellington

Marianne Devere, ESOL teacher, and Monica Luxford, mathematics teacher,

Wellington Girls' College

Abstract

This small study examines language factors in senior secondary mathematics for students who do not have English as their first language. It involved 13 Year 12 and 13 Chinese students at Wellington Girls' College. Parallel tests in English and Chinese were given to the students and in-depth interviews took place. The data confirm a larger study at Auckland Girls' Grammar School (*see* Appendix 1). Students underestimated their language difficulties when self-reporting. The interviews gave insight into the nature of their language difficulties and the strategies the students thought would help their learning.

Background

Elaine Vine heard about the TLRI research at a conference presentation by Pip Neville-Barton late in 2003. Elaine contacted Pip and Pip invited her to attend a team meeting at Unitec New Zealand on 25 March 2004. At that point, Elaine was exploring the possibility of doing some research with teachers at Wellington Girls' College that might be linked in some way with the TLRI research. As it turned out, an Auckland school withdrew from the TLRI project late, and NZCER agreed for Wellington Girls' College to join the TLRI project.

Since we joined the TLRI project late, there was not time for us to develop our own project materials, but the other school teams in the project generously offered us the use of any of their materials that might be suitable for the Wellington Girls' College situation. We decided to use the materials developed by the Auckland Girls' Grammar School team – a short mathematics test, with parallel versions in Mandarin and English (the test items in the two versions had the same wordings, but different numbers). We also adapted the language questionnaires developed by other project schools to suit our context.

We were represented by at least one of our team at every TLRI project meeting held during the year.

The Study

First we explained our proposed project to the school principal and gained her consent for the study to go ahead. We then had a meeting with potential student participants late in Term 2 to explain the project and seek their consent to participate. The participants sat the two versions of the test in lunchtimes, 2 weeks apart, early in Term 3. We would prefer to have had the two testing sessions further apart, for example, one in Term 2 and one in Term 3, but we did not have enough lead time to set that up. We also needed to complete the testing early in Term 3 because these senior school students had heavy assessment commitments later in the term. A few students who had agreed to participate were absent on each testing occasion. In those cases, the students sat the test at another time that was convenient for them. Half of the students were randomly assigned to do the Mandarin version of the test first and the other half did the English version first.

A month after the second testing session, we conducted individual interviews about the test with each of the students who were willing to be interviewed. The interviews were held after school.

Eleven of the 13 students agreed to be interviewed. The interviews were held after school and each student was interviewed in English by one of the researchers — either Marianne or Elaine. We pointed out to the student that the interviewer was a language teacher, not a mathematics teacher. We hoped this would reassure them that we were not judging their mathematics. It also removed any expectation the students may have had that the purpose of the interview was to teach them mathematics. We explained that its purpose was to explore how they went about doing the test items and what role language played. The interviews were audio and video taped with the students' permission. The video camera was trained on the test paper being discussed, not on the interviewee. The videotapes proved to be very helpful when we reviewed the interviews because they allowed us to see where participants were pointing on the test papers as they spoke.

The interviews ranged in length from 26 minutes to 57 minutes. We had asked the students to participate in an interview that would take up to 30 minutes. After 30 minutes in each interview, we pointed out the time to the student and offered her the option of stopping. Most chose to continue the discussion.

Results

Test Results

Thirteen Year 12 and 13 students sat the two tests, and 11 of them agreed to be interviewed. All but one of the students spoke Mandarin as their first language, and all were taking Year 12 or Year 13 mathematics courses.

One student (Student M, Table 1 in Appendix) was from Singapore where her schooling had been through the medium of English. She had been in New Zealand for 6 months. The other 12 students were from China. Three had been in New Zealand for 10–12 months, six for 15–18 months, and three for 2–3 years. All had learned English at school in China, for periods ranging from 6 months to 8 years. None had learned mathematics in English before coming to New Zealand. The number of students in our study was very small, so all of our findings and interpretations must be treated as suggestive rather than as definitive.

The students completed a brief written questionnaire at the beginning of the first test session. They were asked to rate the difficulty of a series of aspects of mathematics and language. The average rating is given in brackets after each aspect below. The rating scale had four points: 1 = not difficult; 2 = a little difficult; 3 = difficult; 4 = very difficult. The aspects are ordered below from least to most difficult as indicated by the average ratings.

Understanding the English used by other Chinese students	(1.2)
Reading the whiteboard	(1.6)
Understanding the English used by my mathematics teacher	(1.8)
Reading mathematics textbooks	(1.8)
Reading the photocopies and handouts my mathematics teacher gives us	(1.9)
Understanding the English used by kiwi students	(2.0)
Reading the mathematics assessment and exam questions	(2.0)
Understanding the English used by other language students	(2.1)

We were surprised that the average ratings were all in the "not difficult" to "a little difficult" range. Only five students rated anything as "difficult" and only two of those five rated anything as "very difficult". The students appear to be reasonably confident about coping with English in their mathematics studies.

However, that confidence does not appear to be well-founded if we look at their performance on the two versions of the mathematics test.

The one student from Singapore whose schooling was in English scored 14/17 on the English version of the test and 4/17 on the Mandarin version. These were not surprising results.

The 12 students from China (Students A–L, Table 4 below) averaged 36 percent on the English version of the test (with scores ranging from 2/17 to 9/17) and 48 percent on the Mandarin version (with scores ranging from 4/17 to 12/17). These data suggest that there is an overall disadvantage of 12 percent for these students when doing mathematics in English as compared with doing mathematics in Mandarin.

Table 4 **Student scores on two versions of mathematics test**

Student	Months in NZ	1st test	Mandarin test /17	English test /17	Score difference
A	10	English	12	5	-7
B	12	Mandarin	5	5	0
C	12	English	10	7	-3
D	15	English	9	4	-5
E	15	Mandarin	10	8	-2
F	18	English	10	8	-2
G	18	English	5	2	-3
H	18	Mandarin	9	8	-1
I	18	Mandarin	4	4	0
J	31	Mandarin	7	8	+1
K	33	English	9	6	-3
L	36	Mandarin	8	9	+1
M	6	Mandarin	4	14	+10

Mandarin test first:

Average score on Mandarin test 42% (range 4/17 to 10/17)

Average score on English test 41% (range 4/17 to 9/17)

English test first:

Average score on English test 31% (range 2/17 to 8/17)

Average score on Mandarin test 54% (range 5/17 to 12/17)

The six students who did the Mandarin version first varied in their performance on the English version: two did better on the Mandarin version, two got the same scores on both versions, and two did better on the English version. Interview data suggested that the English scores for these students may have been inflated from doing the Mandarin version first. It is interesting to note that their performance appears to correlate with the length of time they have been in New Zealand. The four who did the same or better on the Mandarin version had been in New Zealand for 12–18 months, while the two who did better on the English version had been in New Zealand for 2–3 years. For all six students, the difference between their two scores was small, but suggestive. It appears that working in Mandarin first may be most helpful in a student's first 2 years in an English-medium mathematics programme, and/or that after 2 years in New Zealand, a student's Mandarin mathematics skills may be weakening.

The six students who sat the English version first all did better on the Mandarin version, regardless of how long they had been in New Zealand. Also, their average score on the Mandarin version was the highest of the four test results (54 percent). It is possible that the familiarity with the test that they gained from doing the English version helped them to do better in the Mandarin version. It is also possible that, as a group, they were more proficient in mathematics than the group who did the Mandarin version first. The latter seems the more likely explanation, because it is hard to see how doing badly on a test in English would be much help in doing a similar test in another language at a later date, which is what the first explanation would suggest. If this group was more proficient in mathematics, then the fact that their average score on the English test was worse than the average score for the other group suggests that the assistance that the Mandarin-first group got from working in their own language first was important.

When we looked at the performance of the 12 students from China on each item in the two parallel versions of the test (see Table 5 below), we noticed some interesting points.

Table 5 **Correct responses on two versions of mathematics test for Students A-L**

Item	# correct on Mandarin test	% correct on Mandarin test	# correct on English test	% correct on English test	% difference
1	3	25%	1	8%	-17%
2	2	17%	1	8%	-9%
3	5	42%	10	83%	**+41%**
4	7	58%	6	50%	-8%
5	5	42%	7	58%	+16%
6	10	83%	5	42%	**-41%**
7	7	58%	3	25%	**-33%**
8a	8	67%	6	50%	-17%
8b	2	17%	0	0%	-17%
9a	10	83%	8	67%	-16%
9b	11	92%	11	92%	0%
10a	1	8%	2	17%	+9%
10b	2	17%	2	17%	0%
11	5	42%	3	25%	-17%
12	8	67%	1	8%	**-59%**
13	7	58%	6	50%	-8%
14	5	42%	2	17%	-25%

There were three items where the students did noticeably better (a difference of more than 30 percent) in Mandarin than in English:

#12 *An isosceles triangle has a base that is three-quarters the length of one of the equal sides. If the perimeter is 33cm, find the base.*

8 correct in Mandarin, 1 correct in English

In interviews about #12, some students said that they didn't know the word "isosceles" so did not attempt this question in English. One student read "three-quarters" as $3^1/_4$.

#6 *When making a milkshake, I use 3 times as much ice cream as syrup and 6 times as much milk as syrup. In my milkshake, how many times as much milk as ice cream do I have?*

10 correct in Mandarin, 5 correct in English

In interviews about #6, several students commented that they found this question difficult to understand and/or confusing in English.

#7 *The perimeter of a rectangular playing field is 140m. If the length of one side is 30, calculate the length of the diagonal of the field.*

7 correct in Mandarin, 3 correct in English

In interviews about #7, some students said that the words "diagonal" and, to a lesser extent, "perimeter" were a particular problem for them in doing the English version of this item.

There was only one item where the students did noticeably better (a difference of more than 30 percent, see Table 5 in appendix) in English than in Mandarin:

#3 *What is the gradient of the line with equation* $y = 4x - 20$*?*

10 correct in English, 5 correct in Mandarin

One student commented that she did #3 first in Mandarin and didn't know the Mandarin word for “gradient”, so asked her friends after the test. They explained it to her and she was able to do #3 in the English version later. Other students also said that they didn't know "gradient" in Mandarin, and some said that they had learned it in English mathematics class in New Zealand.

Interviews

In each interview, we began by asking the student how she found the two tests—easy? difficult? This generated general comments about the tests and doing mathematics in Mandarin and in English. Then we moved on to looking in detail at the student's test papers in the order she did them. We looked first at items the student got right in one version of the test and wrong in the other version, then items wrong in both versions, then items right in both versions, as time allowed. We asked the student to tell us how she went about doing each item, and we discussed any issues that arose.

Towards the end of each interview, we checked with the student whether she knew words and phrases which occurred in the English version of the test but which had not been identified in discussion of particular items; for example; syrup, cliff, base, angle of depression/elevation, discriminant, mathematical statement, gradient, no more than. Both discussion of the test items and this checking of words and phrases showed that the students either did not know or were not sure of the meaning of most of the words or phrases we asked them about.

We noticed the following from the interview data.

Most of the students had noticed that the two tests were parallel versions, and commented that this sometimes helped them when they did the second test (especially if they did the Mandarin version first).

Students had different strategies for attempting mathematics questions. Some attempted to decode each word and if they came across a word that they didn't recognise or didn't understand, they gave up on the question. They also gave up on the question if they found it too hard or confusing, even if they "knew" all the words. Others tried to work out what role an unfamiliar word was playing in the context, or made a guess, and continued on with solving the problem.

The one student from Singapore reported the same sorts of difficulties as the others—unknown words, daunting wordy questions, word order within sentences—but for her the difficulties were in Mandarin, not in English.

Students sometimes showed a narrow understanding of a concept, for example "quarter" in #12 understood in terms of time (= 15 minutes) only, "gradient" in #3 triggered a process for calculating the gradient, but no apparent understanding of the concept.

Students said they made mistakes through inattention, for example, thinking a question was "Does the diver enter the water?" rather than "*When* does the diver enter the water?" They also said they sometimes made mistakes through calculation errors rather than through misunderstanding.

The students said that they did word problems at school in China, just as they do in New Zealand. Several commented that they found longer, wordy questions daunting in English. Some students commented that they had done some of the mathematics in the tests a long time ago in China, and they had forgotten how to do some of it.

We finished each interview by asking the student the following questions:

- What things do mathematics teachers do that help you to understand mathematics in English?
- What things could mathematics teachers do which would make it harder for you to understand mathematics in English?
- What could mathematics teachers do to make mathematics easier to understand in English?

Teacher activities that students noted as being helpful included:

- teachers speaking slowly;
- teachers drawing attention to new words;
- teachers explaining words;
- teachers using examples;
- teachers explaining examples, especially detailed, step-by-step explanations;
- teachers avoiding using novel everyday words when explaining new mathematical terms;
- teachers using simpler terms in explanations;
- teachers giving examples for students to work on;
- teachers avoiding discussing issues using only words;
- teachers supporting explanations with diagrams or other visual aids;
- teachers writing examples and the process for solving them in full on the board;
- teachers giving model answers for assessments;
- teachers being accessible to students for questions;

- teacher coming over when student has hand up, then explaining one-to-one (teachers speak more slowly and use simpler English in this situation);
- teachers not ignoring non-native English-speaking students;
- teachers checking to see if non-native English-speaking students need help;
- teachers making time for students to see them out of class—it can be difficult and embarrassing for students to ask questions in public.

When commenting on teacher activities, some students also volunteered strategies of their own that helped them understand mathematics in English. These included:

- studying alone;
- working with other Mandarin-speaking students;
- asking a Mandarin-speaking friend if unsure of words or concepts (several students said they would do this first, before asking a teacher);
- relying on friends for help;
- using a bilingual (usually electronic) dictionary;
- asking the teacher if unsure of words or concepts;
- asking a relative if unsure of words or concepts;
- transferring knowledge from other curriculum areas;
- putting essential information in a problem into a picture or diagram, or into mathematical statements;
- learning vocabulary in context, i.e. through doing mathematics exercises rather than from word sheets.

We have not attempted to quantify how many students mentioned each activity or strategy in these two lists, because while some students did not volunteer any strategies of their own or activities of the teacher that helped them understand mathematics in English, others were very clear about what helped. Where they did not volunteer any or many activities or strategies, we asked them about suggestions made by others. There was general agreement across the group about the activities and strategies listed above, even on apparently different strategies such as studying alone and working with others. The consensus appears to be that the listed activities and strategies can all be useful in the course of learning mathematics in English.

Discussion

We are concerned that schools and teachers have assumed that mathematics courses are always an appropriate option for international students, particularly from Asia. It appears to be commonly believed that learning mathematics is not as dependent on language skills as learning in other curriculum areas. Our research suggests to us that this belief is not well-founded, and that schools need to reconsider (a) whether international students should automatically be placed in mathematics courses, and (b) if they take mathematics courses, what would be appropriate types and levels of support for them both in terms of language and of mathematics.

It appears that doing the mathematics test in Mandarin first resulted in better performance on the English version, particularly for students who have been New Zealand for 18 months or less. This suggests that providing examples and explanation of mathematics concepts in students' first language could be a very useful support in helping them understand what the English they are working with means.

We found that some students seemed to be much more aware of their own learning strategies and processes than others. Also, some students had a much greater range of strategies at their disposal than others.

We were surprised at the extent to which the students appeared to underestimate the difficulties involved in learning mathematics in English. Some of the students were obviously surprised at the difficulty they had when they tried to explain their thinking in English in the interview situation. If they are unaware that there are problems, they are probably even less aware of the nature of the problems.

Summary

Thirteen senior secondary mathematics students at Wellington Girls' College sat two versions of a short mathematics test, one in English and one in Mandarin. Their performance was not strong on either version, but there was an overall disadvantage of 12 percent on the English version. Questionnaire results suggested that the students underestimated their language difficulties in studying mathematics in English. In-depth interviews revealed that both understanding the English test items, and explaining their thinking in English was difficult for the students, somewhat to their surprise. The students suggested a range of strategies and teacher activities that they found helpful. Some showed much greater awareness in these areas than others.

Appendix 3: Macleans College Study

A Comparison of English First Language and English as an Additional Language Mathematics Students

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Abstract

This study compared two groups of Year 13 students, one of Chinese-speaking students, and the other English L1 students. The two groups were matched on their mathematical ability as represented by Year 12 grades. Both groups were given a test of Year 12 level mathematics that was designed to explore responses to different discourse features, for example whether or not couching a question in context makes a difference. They were also asked to self-report their level of understanding of English and little difficulty was reported. Eight of the 32 questions showed significant differences in the responses of the two groups. These appeared to be caused by particular English phrases, and no generalisations about types of questions causing difficulty, or vocabulary vs syntax could be made.

Background

Macleans is a co-educational, decile 10 state school. In 2004, the school had a roll of approximately 2,300. The school roll had been predicted to stabilise around the 1,850 mark but has risen owing to in-zone migration and many students postponing their decision to leave school in the current economic climate. The school is divided into eight whänau (schools within schools) of approximately equal size. It has a distinctive ethnic composition with some 50 percent of European origin and the vast majority of these being of British ancestry with an increasing proportion from South Africa. The actual breakdown is given in Table 6 below. There is an increasing percentage of students of Asian origin from a relatively uniform social and economic group, the upper middle class. Families have high academic expectations reflecting the educational opportunities offered to them by parents keen to see their children succeed. This is illustrated by the school's TOSCA median, which is in excess of 60. Streaming occurs for two classes in both Year 9 and Year 10. International fee paying students are a relatively new feature of the school and account for 5 percent of the school roll.

The following statistics indicate the multicultural nature of the school:

Table 6 **Ethnic make-up of Macleans College**

Distribution of students by ethnicity	
New Zealand European	34%
New Zealand Mäori	1%
Korean	10%
Indian	5%
Taiwanese	6%
Middle Eastern	2.5%
Other European	13% (mainly South African)
Chinese	21%
British	2%
Others	5.5%

The authors have experience in teaching mathematics to EAL students of Asian descent, EAL students of non-Asian descent, and English first language students. We would describe the EAL students of Asian descent as, typically, mathematically able, algebraically literate, and relatively studious by comparison to the average New Zealand student. Language difficulties for these students vary with length of time they have spent in New Zealand schools. They are generally not responsive to oral questions in class. There is a high correlation between their performance in English and mathematics. The very best of these students mathematically have a high ability to communicate mathematics in English.

Our expectations of the results of this study were that the Chinese-speaking students would have markedly less success with wordy problems. We anticipated some specific vocabulary difficulties.

The Study

A test was given to all Year 13 statistics and modelling students (135 students). The students cover the full range of ability levels, reflected by their grades at 6th Form mathematics. However, this group excludes the group of most capable students at Year 13 who are doing the Cambridge International Exams (80 students).

With the test we collected the following demographic information:

- student's first language;
- country where schooling began;
- number of years in New Zealand;
- number of years of learning mathematics in English;
- grade in 6th Form mathematics;
- personal perception of degree of difficulty in understanding written and oral mathematical discourse in English.

Skills that were tested included the following:

- understanding of syntax;
- specific vocabulary – both recognition and recall;
- ability to extract the mathematics from a problem in a practical context;
- ability to extract the mathematics from a problem with redundant information and wording.

The specific mathematical content in the test was all at a Year 12 level. The specific mathematical topics covered were:

- finding the midpoint of line;
- finding a derivative;
- finding the distance between points;
- description of graphs;
- algebra.

The first three topics listed above were tested in three or four separate questions. Each time the question was posed a different way, for example:

- using a primarily visual or graphic approach;
- using a simple mathematical expression with as few words as possible;
- in a straightforward word problem;
- in a word problem in a practical context;
- in a word problem in a practical context containing redundant information.

Our initial analysis of the answers excluded questions 10 and 12 which are about vocabulary recall. Questions were marked correct (1 mark) or incorrect (0 marks). In some cases the question was broken into two or more parts and each part marked in this manner (there were 32 parts in total). Minor computational errors were ignored where it was clear that the student had understood the question and knew how to proceed to answer it. The results were entered into an Excel spreadsheet.

We discovered while the test was being administered that question 14 was mathematically flawed and it has therefore been excluded from the study.

We identified 68 English first language speakers and 38 Chinese first language speakers by their answers to the demographic question on first language. We refer to them as Group E and Group C. The remainder were from a variety of backgrounds, the largest sub-group being Korean. Looking at the results, on the surface Group C outperformed Group E on 16 of 32 parts, contrary to our expectations. However, when we controlled for mathematical ability, as indicated by 6^{th} Form Certificate grade point score, Group C outperformed Group E on only nine of the 32 parts and equalled them on two parts.

The average 6^{th} Form grade point score was 3.4 for Group E and 2.4 for Group C, indicating significantly higher mathematical ability for Group C on the whole. We controlled for this by selecting only those students with 6^{th} Form grades of 2 and 3. We also wanted to exclude members of Group C who had many years of mathematical instruction in English and therefore selected only those members of Group C who had 5 years or less of mathematical education in English. The reduced Group E has 17 members and the reduced Group C has 14 members. The two reduced groups have identical 6^{th} Form grade point averages of 2.7. The results in the balance of this report refer to these reduced groups. Note that, if the hypothesis of this study is correct and lower English proficiency does affect mathematical achievement, then Group C is probably better in mathematical ability.

For each question or part question we calculated the proportion of correct answers for Group C and Group E. We then calculated a z statistic comparing the two proportions:

$$z = \frac{p_E - p_C}{\sqrt{\frac{p_E(1-p_E)}{n_E} + \frac{p_C(1-p_C)}{n_C}}}$$

Four of the 32 z values tested were significant at the 5 percent level and a further three were significant at the 1 percent level. A two-tail test was used because the z values could be either positive or negative.

Results for questions 10 and 12 were analysed in the same manner. Each item of vocabulary was given a mark of 1 if it was mathematically and grammatically correct when inserted into the blank in the question, and otherwise 0. The difference between Group E and Group C for the seven parts in total was not significant. Only one of the seven parts showed a significant difference at the 5 percent level.

Results and Discussion

Overall, Group C students reported only “a little difficulty” with understanding the English language of their mathematics classes. Only five of the 14 students reported “some difficulty” on any one of the four aspects they were asked about (teachers’ language; reading texts; reading exam and test questions; understanding other students).

The eight parts showing significant differences between Group E and Group C are discussed below.

Question 1(a) Part 2 (Difference in favour of Group E; 5 percent significance).

Subtract the result from 16.

A significant number of students of Group C incorrectly interpreted this question. Confusion lay in the ability to distinguish the difference between doing as stated as opposed to subtracting 16 from the result, that is they have reversed the intended operation. Perhaps we can conclude that many students rely on key words, such as “subtract” and “16” and the order implied by the sentence was not well understood.

Question 1(a) Part 4 (Difference in favour of Group E; 1 percent significance)

Add the answer to 60.

The question seems without difficulty but was surprisingly poorly done by group C. Many interpreted the question to mean "What number must be added to **get** 60?" For example, if the preceding answer was 10, then 50 was given for the new answer. This part in particular uses simple words with no obvious ambiguity to English speakers. Either the word order, or the preposition, caused difficulty.

Question 1(a) Part 8 (Difference in favour of Group E; 5 percent significance)

Divide your answer into 100.

While this was poorly done by most students of both groups, again Group C performed relatively badly. The common mistake, as with Question 1(a) Part 2 is the order of the operation was reversed.

Question 1(b) Part 4 (Difference in favour of Group E; 1 percent significance)

Students were asked to identify descriptive forms of $\frac{3(3x+4)}{x^2}$.

Poor identification of this possible answer indicated confusion between dividing by and dividing into. Another option that was written as the symbols are written, that is, numerator first followed by the denominator, was well done by most students.

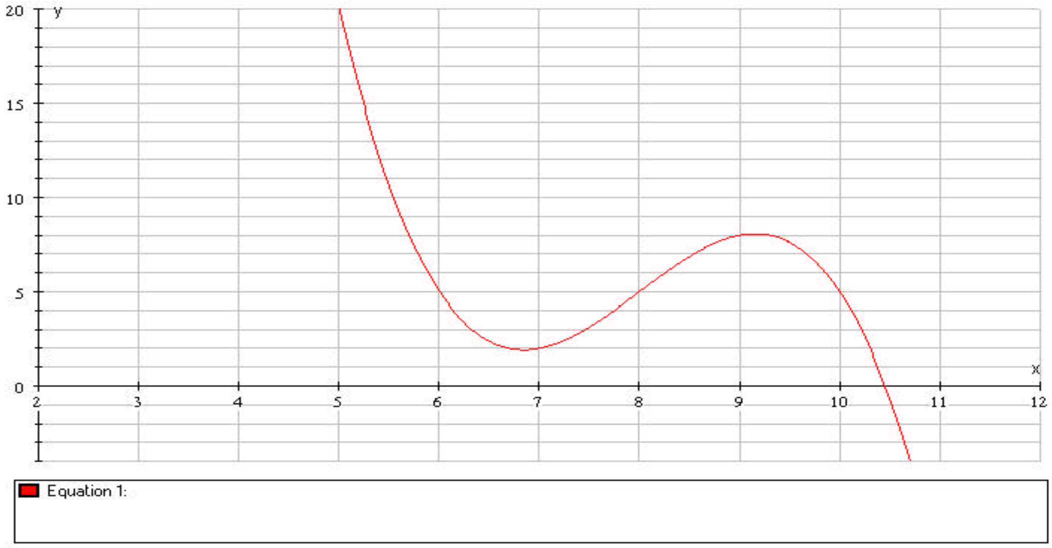
Question 7 (Difference in favour of Group E; 1 percent significance)

Ship A is 8km West of a rock and 10km South. Ship B is 4km North and 5km East of the same rock. What is the shortest distance between the two ships?

This question required interpretation of geometrical information in plotting coordinate points. Group C encountered significant difficulties with this, perhaps due to inability to turn written instructions into a visual diagram to aid solution. This is in contrast to similar skills required in question 5 in which no significant difference was noted when the coordinates to be used for the calculation were explicitly given.

Question 8 (Difference in favour of Group E, 5 percent significance)

Using the curve below, estimate the gradient of the curve when x =10.



Both groups performed poorly on this question but Group C performed significantly worse. This question proved demanding for both groups and essentially tested conceptual understanding and involved minimal English. The language was straightforward with the only vocabulary of any difficulty being “estimate” and “gradient”.

The answers by Group C showed a variety of errors and several made no attempt at an answer. Possible reasons for this may be:

1. New style of question. Again this is true for both groups. Perhaps the learning of Group C relies more on having seen or completing similar written questions as opposed to English-speaking students listening, understanding, and recalling verbal instruction or concepts that have had minimal repetition in class or at home.

2. Students focus on a calculation type response and either ignore much of the verbal conceptual understanding and background, or do not have a strong background in concepts through reading significant portions of the text The students were more than competent in dealing with the evaluation of a gradient from an algorithmic standpoint (questions 4 and 6) so it may be assumed that they do not recall explained concepts in class as well. We believe that much student learning is achieved through practise of problems at home by themselves as opposed to being complemented by teacher instruction and help. In other words much of what is being said in class is either ignored, is not understood, or is not viewed as important. This hypothesis needs to be further researched.

Question 9 (Difference in favour of Group C; 5 percent significance)

Find the coordinates of the midpoint of the line segment joining (3, 1) and (8, 7).

This is the only question with a significant difference with Group C outperforming Group E. Given that the groups have similar ability in mathematics one might speculate that this result is due to Group C students being more methodical and careful in calculations.

Question 12 Part 1 (Difference in favour of Group E; 5 percent significance)

This is a graph of a sin curve. It has an ________________________ of 3.

The correct answer was "amplitude". Three of the Group C answers which were marked as wrong, were close to correct and would possibly have gained a mark in an exam situation. In this situation the difference between the two groups would not be significant.

Other Results

When we compare the performance on the questions which were stated in simplest terms (questions 4, 5, and 9 taken together) against the performance on the wordier and context-based questions (questions 7, 11, 13, 16, and 17) we find that both Groups E and C performed worse on the latter questions, and that for both types of questions taken together the difference in performance between groups is significant at the 1 percent level in favour of Group E. This result is to be expected. However, we had anticipated that Group C would do relatively worse than Group E on the wordier and context-based questions and this has not been borne out by our results. There is little difference in the relative performance over both types of questions.

The tests of recall of specific mathematical vocabulary also did not produce significant differences, contrary to our expectations.

Summary

Our study compared groups of similar mathematical ability in attempting to identify language difficulties.

The group of EAL students (Group C) did not self-report any serious difficulty with the English of their mathematics classes.

The difficulties of Group C appear to relate to specific turns of phrase in English that are not well understood. Word order and prepositions caused problems, but there was no evidence for making further broad generalisations, such as that wordier or context-based questions, or memorising vocabulary, cause more difficulty for EAL students. Group C may also have greater difficulty than their English-speaking counterparts with questions posed in a form that they have not encountered previously.

Our work could be extended in several directions. Further research should be done, first to attempt to replicate these results, and then to take the additional step of interviewing individual students to attempt to confirm their understanding of the questions and the specific sources of difficulty.

Our results do not necessarily extend to the more mathematically able and less mathematically able students. It might be valuable to conduct this research with groups of students in these categories to determine the sorts of difficulties they encounter. Similar caveats apply to the age of the students tested.

Appendix 4: Tangaroa College Study

Language Factors that Affect Mathematics Teaching and Learning of Pasifika Students

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Abstract

This study investigated the language issues for senior Pasifika mathematics students in one school in the Manukau region. Using the experiences of the Pasifika teacher/researchers, a test was constructed to investigate likely language difficulties. From these results a second test was conducted specifically investigating word problems. Some interviews were conducted about the tests. Initial perceptions that vocabulary was the main problem were not borne out in the first test, although the second test and interviews indicated that the language issues were interwoven. There appears to be a difference between Tongan and Samoan students, and between those who knew their Pasifika language well and those who did not.

Background

The emphasis that has been put on students' ability to articulate their strategies, discuss ideas and concepts critically, and communicate mathematical meaning has become a more central focus in mathematics, and mathematics education. The teaching pedagogies that are required to successfully achieve these purposes have put pressure on students from any group that does not have English as their first language.

The steady increase in bilingual learners is a reality of New Zealand classrooms, particularly in the Manukau region. In many of these classrooms there are students with different levels of competency in the language of instruction, and few—if any—teachers are able to speak to these students in their mother tongue. In addition, English is not the mother tongue of many of the teachers. The teaching and learning of mathematics in a language that is neither the teacher's nor the pupil's main language places additional and complex demands on teachers and learners.

The overall aim of the research project of which this study is part is to examine the nature of language factors in the learning of mathematics for students for whom English is an additional language (EAL) in order to make recommendations for mathematics teachers of EAL students, and provide guidelines for the design of language support programmes. This part of the project focuses on Pasifika-speaking students. It uses the personal experiences of the researchers to identify language features that might cause difficulty, and then tests whether these difficulties are widespread in a large population of senior secondary Pasifika students.

All the four teacher/researchers are from the Pacific Islands. The two female teachers are from Samoa; one was born and raised in Samoa, while the other was born in Samoa but moved to New Zealand at a young age. The other two teachers were both born and raised in Tonga. They moved to New Zealand at senior secondary school level. In our initial discussions about some of our experiences, we concluded that our experiences are broadly typical of the students in the study, and the conflicts we experienced are conflicts that many migrant Pasifika students experience today. We discussed some of the problems that we faced when we were students and also reflected on our experiences as teachers. It was our prior perception that Pasifika students do have difficulty learning their mathematical vocabulary.

The Study

The two classes being studied were Year 12 classes from Tangaroa High School, which is situated in the Manukau region. The Manukau region is New Zealand's most multicultural area with the highest percentage of Pasifika peoples. All students were non-European, but they had different levels of competency in their own languages. Most of the students came from homes where the first language was not English. The interview data show that the majority of these homes are involved in community and church activities that use their mother tongue as the language of communication.

The teacher of Class A is a Tongan male teacher who can speak Tongan fluently. There were five Tongan students in his class, but only three of these students were fluent in the Tongan language. The teacher of Class B is a female Samoan teacher who can speak Samoan fluently. The majority of her class (24 out of 28 students) were Samoan students and only two of these students were New Zealand born while the rest are recent migrants.

In Phase 1 of the study, the team leader visited the two classes on a weekly basis for two terms. Together with the class teachers, he looked closely at individual students' work as they learned mathematics, and listened carefully to their conversations noting what languages they used the most, who they talked to, when they used English, and when they chose to use their mother tongue. Observations were also made on what language the teachers were using.

This information, and discussions amongst the researchers about their own experiences, led to the construction of the first questionnaire. It was aimed primarily at testing students' knowledge of vocabulary: words used in mathematical instructions, technical vocabulary, and vocabulary used within a word problem situation. Test instructions were written in both English, and either Tongan or Samoan. Students were also asked how they felt about their level of understanding of English in mathematical learning. Phase 2 of the project was the administration of this questionnaire to both classes.

A main result from the first questionnaire (see below) was that students have difficulty solving mathematical word problems. Even though this result is similar to students for whom English is their first language, we felt it would be useful to look for specific examples of language features that cause these problems for Pasifika students. The second questionnaire, therefore, was constructed of word problems, each of which used one of the following characteristics: conditionals; inequalities; implications; and negations. All questions were written in both English and either Tongan or Samoan on this questionnaire. In the third phase, this was administered to both classes.

An interview schedule was designed to gather more information on which language students used most often when they were at home, at school, and in the wider community. It also included questions about the difficulties they have when they work on word problems. Four word problems were taken from the second questionnaire, one from each of the following categories: conditional; inequalities; implications; and negation. Sixteen students drawn from volunteers from both classes were interviewed. Two were interviewed individually, and the other 14 in pairs.

Results and Discussion

Phase 1: Classroom Observations

Discussing mathematics was not a normal classroom practice for the students in these two classes. There was no use of group investigation, problem solving, group discussions, or hands-on type activities. Students were expected to work quietly and individually. The two classes did not have a textbook, although photocopied work sheets were given out each day. Students were exposed to limited language forms: the classroom displayed little mathematical language, and the classroom seating organisation did not promote the possibility of inter-student communication.

In both classes, code switching was a common practice between students with the same mother tongue. Students tended to use mostly their first language in their personal conversations and also in mathematical conversations when these occurred. In most cases, this happened immediately after the teacher's instructions for the whole class. On many occasions, the teacher of class B switched from English to Samoan when she moved around to help individual students in order to explain and clarify the mathematical concepts. A group of Samoan students in class B always conversed in Samoan among themselves, and their expectation of the teacher was to reply in Samoan when they asked questions. The teacher's view on code switching was:

> In the classroom, when I am explaining a concept to a Samoan student, I switch from English to Samoan and vice versa. With students who speak very little English I don't even have to think about the language I use—that being Samoan. However, with Samoan students who are fluent English speakers, I explain it in English and then if necessary in Samoan. My thinking behind this is to speak in the language the child is most comfortable in to aid understanding.

The teacher wanted the students to understand the mathematics, and recognised that they needed to use their main language in class. The teacher's beliefs made it comfortable for students in her class to switch.

Phase 2: The First Questionnaire—Vocabulary

The first questionnaire was divided into five different sections: Self-Reporting English Competency; Mathematical Instructions; Mathematical Vocabulary; Mathematical Language; Word Problems.

Self-Reported Competency in English

The responses show that over two-thirds of these students feel that they have no, or only a little, difficulty with reading mathematics textbooks, handouts, and test questions.

Table 7 **Self-reported competency in English**

Numbers of respondents (n = 42)	No difficulty	A little difficulty	Some difficulty	A lot of difficulty	Not sure
Reading mathematics textbooks	17	11	10	3	1
Reading mathematics handouts and test questions	15	14	8	3	2

Instructional and Mathematical Vocabulary

The following six instructional words were tested: solve, evaluate, simplify, factorise, expand, and rearrange. Different mathematical solutions were given together with the instructional words, and students were asked to choose the word that best described the given working.

It was expected that the students would not have a good enough understanding to make the match. However, the overall results indicate satisfactory understanding of all terms except “evaluate”. There were two questions that were given to test their understanding of “evaluate”, one with a function and the other with an equation. More students got the right answer when they were given an equation than when they were given a function. More students mistakenly chose “rearrange” as their answer when a function was given. There was an understandable confusion between “factorise” and “simplify” as the procedure showed a simpler equation after the factorisation process. The other common mistake was with the word “solve”: students seem to choose solve as the answer for any working that ended with a number value.

Students recognised the expanding procedure when two brackets were given, but did not do well when there was only one bracket. Students might have only been exposed to expanding when they worked with quadratic expansion. This is evidence of restricted meanings where new vocabulary is associated with the exact context in which it is learnt and not a more general concept.

Mathematical vocabulary was generally better understood, except for some of the more highly technical descriptions of functions (see Table 12 below). Students found the question that tested their knowledge of “simultaneous equation”, quite a challenge. This may be due to their unfamiliarity with the words, and perhaps the unfamiliar way in which the question was asked.

Word Problems and Mathematical Language

Word problems proved difficult for these students, even when the working was provided. The results for two multichoice questions are given below.

> Sione’s mother, Ana, is six years older than Sione’s uncle, Tevita. Tevita will be 60 years old in two years time. Ana’s age, **a**, is:

Table 8 **Summary of results for question 4a**

	a = 60 + 6 – 2	***a*** = 60 – 6 – 2	***a*** = 60 + 6 + 2	***a*** = 60 – 6 + 2
Percentage of students	45 v	0	23	33

A number **n** is the product of itself minus 6 and half of itself.

Table 9 **Summary of results for question 4b**

	n = (***n*** – 6) + ***n***/2	***n*** = (***n*** – 6)***n***/2	***n*** = (***n*** – 6)/2	***n*** = ***n***/2 – 6
Percentage of students	13	31 v	46	10

Although it is made up of one sentence, it was difficult for students to construct a formula for the "number" ***n***. The complexity of working with a variable and the meaning of "product" together with the reference to the number "itself" all contributed to the difficulty.

An example of a problem illustrating the students' difficulty in interpreting a situation was the following:

> A wall is 2m high at one end, 3m high at the other end, and has a length of 6m. Draw the wall in the space below. The area of the wall is: (i) 15m square meters, (ii) 30m square meters, (iii) 30 square meters, (iv) 15 square meters.

Table 10 **Summary of results for question 5b**

	15m sq.m.	***30m sq.m.***	***30 sq.m.***	***15 sq.m.***
Percentage of students	11	56	17	16 v

This required the students to draw a diagram that represented a simple composite shape. The problem was poorly done; only 16 percent completed it successfully. Although the students could follow a procedure for drawing a wall, they lacked the ability to sensibly interpret the situation, and apply their knowledge to the problem context. Their interpretation of the length of the wall in most cases did not link to the idea that it is most likely to be associated with a level floor or level ceiling. Hence, their drawing showed two sloping lines between the two walls. This highlights that students must have enough experience to identify various contexts in which everyday background knowledge could be applied.

Comparison between L1 and EAL Students

A comparison between students who claimed that English is their first language (L1) and those who claimed that a Pasifika language is their first language (EAL) reveals some interesting results. Table 11 shows that students whose first language is a Pasifika language perform better overall than those who speak English as their first language.

Table 11 **A comparison of performance on instructional words**

	% of L1 students with correct answer	% of EAL students with correct answer
Evaluate	15	35
Expand	53	91
Simplify	61	56
Solve	61	69
Factorise	46	78
Rearrange	61	69

The results were more mixed for the mathematical vocabulary, as shown in Table 12.

Table 12 **A comparison of performance on mathematical vocabulary**

	% L1 students with correct answer	% EAL students with correct answer
Coefficient	61	87
Numerator	85	74
Denominator	92	78
Exponent	46	70
Linear	98	96
Quadratic	0	100
Cyclic	7	52
Exponential	7	43

All students showed a good understanding of numerator and denominator, with L1 students being better, but the EAL students understood the other terms better. It appears as though the L1 students had trouble with all algebraic terminology.

Phase 3: The Second Questionnaire—Problem Solving

Questions on the second questionnaire were aimed at finding out students' understanding in the areas of logical connections, implications, inequalities, and negations. Questions were presented in both English and either Tongan or Samoan. This meant that the emphasis was on the understanding of mathematical relationships, rather than on the particular structures of mathematical discourse in English alone.

Evidence from students' work showed that students had difficulties with correctly expressing problems in mathematical statements, and interpreting basic mathematical terms.

The first question was *"List all the multiples of six between 546 and 618 inclusively."* A lot of students who could list the multiples did not take extra care to include the two end numbers, 546 and 618. The other interesting result from this question revealed the difficulty that Pasifika students face in mathematics even if they speak Samoan or Tongan. There is no equivalent word for "multiple" in either the Samoan or Tongan languages. There are two approaches for translation. It is possible to base the translation on the divisibility of numbers. In this case one must take into account that the context of this question requires results that give whole numbers. Alternatively it is possible to use the times table, and refer to the product of two factors of which one is the "multiple concerned". The Samoan translator used a "multiplicative" approach whereas the Tongan translator used the "divisibility" approach. It was interesting that five of the

Samoan-speaking students listed all of the factors that can be multiplied by six to get a number between 546 and 618. One student listed all the factors including decimal numbers.

Results from the rest of the questions on the questionnaire are summarised in Table 13 below. Students were required to show their mathematical working for all these questions. However, most of these questions were either left blank, or had answers that had little or nothing to do with the given question. These results suggest that these students had severe difficulties interpreting and effectively solving word problems, and rationalising solutions consistent with the problem's context.

Students' performance shows that "implication" word problems were the hardest for them to solve.

The poor performance on this second questionnaire—which was presented in both English and the student's home language—indicates either low proficiency in both languages, or a lack of comprehension of the nature of mathematical discourse itself (in any language).

Table 13 **Summary of questions 2 to 8**

		Number of students with the correct answer	Number of students with the wrong answer	Number of students not attempting the question
Qn.2.	Question needing the transformation of a described situation into algebra	8	14	11
Qn.3.	Question needing the transformation of a described situation into algebra	8	18	7
Qn.4.	Question involving inequalities	3	20	10
Qn.5.	Question involving inequalities	1	12	20
Qn.6.	Question needing the transformation of a described situation into algebra	1	12	20
Qn.7.	Question involving negation	5	5	23
Qn.8.	Question needing the transformation of a described situation into algebra	4	5	24

Phase 4: The Interviews

The first part of the interview revealed that most of the students were bilingual. However, their fluency in both languages was lower than what could be expected of Year 12 students.

The second part of the interview provided some results about students' understanding of the mathematical meaning of the words used in the word problems. Students were verbally fluent when asked to read a question aloud, although some could not verbalise symbols. When they were asked specifically whether there was anything that they did not understand, they all responded positively about their general understanding. However, being able to read the text or the problem does not guarantee understanding of the concept or process, nor does both reading and understanding imply an ability to solve the problem.

Many of the difficulties students encountered with mathematical word problems were those concerned with relational statements. Relational statements include "transitive inferencing" problems, for example, “Viliami’s salary is less than twice Ana’s salary.” This was problematic for all except one student. When students were asked to write mathematically “twice Ana’s salary” (given that Ana’s salary is A), students found it difficult to use variables until the interviewer gave them hints. Common mistakes were AA and A^2 even when some could say that twice means “double”.

Some students did not know the word for the greater than sign “>”. One student referred to it as “over”, and some missed it out completely when they were asked to read the question aloud. “Less than” was often mistaken with subtraction: four students wrote $V - 2A$ instead of $V < 2A$.

Summary

This study has made us, as mathematics teachers, aware of the differences between the language that bilingual students use in the mathematics classroom and the language that they use in their home and social environment.

The translation of the questions used in the second research questionnaire revealed that both Tongan and Samoan languages have mathematical discourses, but they not yet have been fully developed. Thus bilingual students are unfamiliar with many mathematical terms and phrases both in their first language and in English. The complexities of mathematical sentences have been shown to provide extra challenges for these students’ learning. These factors play a major role in the language features that impede Pasifika students’ learning of mathematics.

Students in this research performed well on questions involving instructional vocabulary. However, when they were given word problems that required them to read a question or statement, think, analyse, and carry out appropriate computations, most students did not have the appropriate problem-solving strategies. This may be partly due to minimal exposure to problem solving in their mathematics class. Students need to be strong in both their general and their mathematical language. These combine to provide the comprehension skills needed to successfully make sense of and solve mathematical word problems. If their mathematical background is poor at this level, it does not matter whether we test them in their mother tongue or in English—either the relational understanding has not been laid down or there is a combination of poor English ability and undeveloped mathematical discourse in their mother tongue. It is our belief that it is more a problem with their mathematical understanding rather than the language of instruction, but this needs more investigation.

There is not enough evidence in this study to either support or argue against the theory that students who use their mother tongue while learning in English perform better than those who do not. It may be a wise investment for these students and Pasifika communities to develop and encourage their children to practise their mother tongue. This is another question needing research.

This study indicates that many of these students overestimate their facility with English. Although Pasifika students appear to be comprehending written and spoken English, this may not be the case, despite the fact that English is a first language for many of the students. For the Pasifika students who claim that English is their first language, research needs to find ways of developing these students’ literacy in English, particularly in mathematical discourse, and to consider what constitutes appropriate support at secondary school level given that there have recently been positive outcomes in primary and intermediate school education. It is our recommendation that teachers should be more explicit with their teaching style, pointing out different language features that are important to their understanding and checking that their meaning is appreciated.

Appendix 5: The University of Auckland Study

The Mathematical Discourse of Advanced Undergraduate Mathematics

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Abstract

Third-year university mathematics courses were observed for the differences in the way language is used compared with first-year classes. Significant differences were found, with mathematical understanding being much more deeply embedded in the language of the lecturer and texts. A small group of Mandarin and Cantonese-speaking students were then tested on specific mathematical items, and also their self-perception of their understanding of the course. English language proficiency results were also available. The results showed that the disadvantage for these English as an additional language (EAL) students due to language is higher than expected, and that they are unaware of their difficulties. It also identified multiple adjectival phrases, some syntactical features, and logical complexity as particular problem areas. A follow-up on a larger class including English first language (L1) students confirmed these results and indicated that L1 students do not have language problems. It is concluded that increasing linguistic complexity at higher levels of mathematics is partial explanation of the declining grades of EAL students compared with L1 students as they progress through undergraduate study in this subject.

Background

Universities in New Zealand are continuing to accept students who have a language background that is not English, the language of instruction. Thus there is ongoing interest in language requirements for tertiary study, and in the provision of programmes that will assist students in their studies. At tertiary level, students with poor English language take mathematics under the impression that they will not be so disadvantaged. Many perceive it to be relatively language free. At the University of Auckland, where well under half the mathematics students have English as a first language, there is a need to know both what levels of English proficiency are necessary for successful mathematics learning at this level, and what measures can be put in place to help students learn mathematics in English.

Investigations into language issues in mathematics education at secondary level and in bridging or entry-level tertiary programmes are limited in number (see Adler (1998) for a review). This paper describes a study at advanced undergraduate level that aims to investigate how students who have English as an additional language (EAL) are understanding mathematics differently compared with students who have English as a first language (L1). What are the discourse features that cause problems for EAL students?

An earlier University of Auckland study, involving 80 first-year students, used text, symbolic, and diagrammatic questions to indicate the extent of textual difficulty experienced by EAL students in first-year undergraduate mathematics (Barton & Neville-Barton, 2003). It indicated that, in comparison with native speakers of English, EAL mathematics students have a disadvantage similar to that experienced in arts subjects (about 10 percent). A second study (involving nearly 400 first-year students) found that EAL students self-reported levels of understanding similar to those of English first-language students (Barton & Neville-Barton, 2004).

This study focuses on advanced (third-year) undergraduate mathematics. We wish to know whether the disadvantage is as marked at this level, and to understand what features of mathematical English cause difficulty. We notice that the proportion of EAL students taking mathematics drops dramatically at this level. There may be other explanations—for example cultural preferences for other major subjects—however we also suspect that there is a change in the nature of mathematical discourse and its relation to the mathematics presented at this level.

There is a considerable body of literature that examines the nature of mathematical discourse. Halliday (1978) is usually credited with focusing researchers' attention on mathematical language as a special register. Dale and Cuevas (1987) describe the mathematics register in terms of unique vocabulary and syntax (sentence structure), and discourse (whole text features). Also mentioned in the literature above are such features of mathematical discourse as its density, logical complexity, heavy demand on reader's memory, unpredictability, and the mix of prose, symbols, and diagrams. While these are postulated as potential sources of difficulty for EAL students, whether in fact they do present problems, particularly at higher levels of mathematics, is the issue being considered here.

The Study

This study involved four third-year undergraduate mathematics courses in the Department of Mathematics at the University of Auckland. Two of the researchers are the lecturers of two of these courses. The study had three phases, two in the first semester and one in the second. The first phase involved one researcher attending lectures of the other two researchers, examining texts and course notes, and trying to identify significant mathematical discourse features at this level of mathematics. In the second phase, these features were transformed into a test/questionnaire that was presented in a tutorial to 12 Chinese-speaking students from one of these courses. The second phase also involved English proficiency testing using the University of Auckland's Diagnostic English Language Needs Assessment (DELNA).

In the third phase the test instrument was re-used with two larger third-year classes (53 EAL students, eight English first language students (L1)). This was done to confirm the earlier results. No English testing was undertaken with this larger group.

Results

Phase 1: Observations of Third-year Mathematics Classes

Significant observations were made under three themes: the role of the lecturer; vocabulary and syntax; and logical complexity.

The Role of the Lecturer

In general, the role of the lecturer could be seen to be different at third-year level compared with first-year. In first-year courses the lecturer usually closely follows a text or course notes, and, more often than not, they perform a calculation or procedure. Thus what they say is usually being written down while they are speaking, or an example of it is available as text. Lecturers describe and explain examples that have a known focus. There is redundancy in the discourse, repeated similar examples, and the style is familiar. Once a student understands what is happening, then they can predict what will be said, and listening becomes a confirming activity.

At third-year level there was less describing and explaining, and more defining, linking, and illustrating. This imposes different demands on a listener. Defining, linking, and illustrating mean that apparently random mathematical topics may suddenly arise in the course of a lecturer's delivery. All the talk is vital, and it may shift focus unexpectedly. Illustrations or examples are usually different from each other in a fundamental way. An example of a theorem is not similar to all other examples, rather each one illustrates another aspect of the theorem. Counter-examples become important.

Vocabulary and Syntax

The technical vocabulary has many of the same characteristics as the technical vocabulary at earlier levels, and students are likely to be familiar with it. However, in addition, there is an increasing use of general English words being used as technical language. For example, in the topology course, it was noted that T_1-spaces are "weaker" than Hausdorff spaces. This use of "weaker" is unusual for two reasons. One is that it may be intuitively opposite to the normal sense of the word. T_1-spaces are weaker because there are *more* of them, whereas weakness is usually associated with less of something. It is also unusual because it does not refer to the *object*, it refers to the separation property that that object represents. It is weaker because of how it is used, not because of what it is or what it does.

A second vocabulary issue that arises with more frequency at this level of mathematics is the use of suffixes to form a technical term, for example "analyticity" and "differentiability". Such constructions are easy for a native speaker, as the meaning is intuitively apparent. For an EAL student the meaning needs to be learned.

The use of multiple adjectives or adjectival phrases also increases at this level. Consider this statement: “Let *u* and *v* be two continuous real-valued functions of two variables having continuous first partial derivatives that satisfy the Cauchy-Riemann equations in some domain D.” There are eight aspects to *u* and *v* that we need to keep in mind from this statement. When discussing this research with a colleague who learned psychology in English as a Dutch speaker, we were told that even after 20 years she still has to translate back to Dutch when she meets two combined words (such as “projective identification”).

In general English, adjectives may be indicative or descriptive. “Look at the green trees on the hill.” “Look at the tallest tree on the hill.” In the first sentence the word “green” is more or less redundant. In the second sentence the superlative “tallest” indicates exactly what is being referred to. In mathematical discourse adjectives are nearly always indicative. Thus every adjective must be comprehended exactly, increasing the memory as well as cognitive load.

Logical Complexity

Mathematical discourse at this level becomes much more dense in its logical structures. For example: “An interior point of a set is a point such that you can construct an open interval on that point which is entirely in the set.” The formal logical structure of the sentence is: “X is a Y such that Y has a Z that is a W.” Not only is the logical structure complex, but X, Y, Z, and W may also be complex entities. Each one might be a concept that is made up of more than one sub-concept as illustrated in the previous section.

This example is from a definition, and thus it can be expected that considerable attention will be given to the complex meaning of the statement. However, consider the following two examples.

This statement occurred in the proof of a theorem: “If $x = \{x_i\}$ and $y = \{y_i\}$ are two distinct points in X, then we must have $x_{i(0)}$? $y_{i(0)}$ for at least one index $i(0)$.” This has the logical structure: “If A and B are C, then we must have D for at least one E.” An equivalent general English statement is: “If the trees and the shrubs in the park need pruning, then there are several hours of work for one of the gardeners.”

This statement occurred in an explanation after stating a definition: “If we denote (x, y) by $x + iy$, where $i^2 = -1$, then we can denote the ordered pair (0, 1) by i so that $i^2 = (-1, 0)$.” This has the logical structure: “If A when B, then C so that D.” An equivalent general English statement is: “If I drive to my sister’s place in the springtime, then I will see the flowers in the fields and feel happy.”

Both these examples were intended to be explanatory. They were not the most important pieces of information being presented at that time, thus students were not given much opportunity to understand the statements. However, when these statements are transformed into English equivalents, the linguistic complexity becomes apparent. Readers are invited to read each of the general English statements again and to identify the most important idea. In the mathematical context, the part of the statement that is the main subject needs to be identified quickly and unequivocally if it is to make sense.

Statements of this type are common at third-year level, but rarely occur in first-year courses. Furthermore they are often transformed into inverse statements, or negative statements, and the truth of such derivative statements is vital to the mathematics.

At advanced levels of mathematics there is an increasing emphasis and sophistication about the use of the words "show" and "prove". The context becomes much more significant: what can be assumed, what results can be used, what explanations are required? Consider the following two requests that were observed close to each other in a lecture:

"Show that if $f(x) = x^3$ then $f'(x) = 3x^2$." Implied (and not stated) in this statement was that a proof from first principles was required.

"Show that if $f(x) = 2\sin(x^3)$ then $f'(x) = 6x^2\cos(x^3)$." This statement implied the use of the Chain Rule.

As in some of the other examples above, students, including EAL students, who have been familiar with earlier undergraduate mathematics courses, will be used to interpreting these statements. But such skills are dependent on considerable familiarity with the type of question being asked. However, also observed in these courses, were requests to demonstrate the truth of a result that had a form unlike anything that had been seen before. For example, early in the topology course, when students had only recently been introduced to the idea of closed and open sets, their text contained theorems such as: "Every compact subspace of a Hausdorff space is closed", and "In a Hausdorff space, any point and disjoint compact subspace can be separated by open sets, in the sense that they have disjoint neighbourhoods." These theorems were on the same pages as definitions of T_1-spaces and Hausdorff spaces. Furthermore, proofs of these theorems included assumptions that were not stated in these pages (for example, that an empty set is open).

In order to be able to provide a suitable proof, the student must know considerable detail about the context of the question. In advanced mathematical courses these contexts are often new, and usually are subtle. They are indicated linguistically by unusual relationships between things being discussed (for instance, in the example above, the objects T_1-spaces and Hausdorff spaces were presented as "separation properties").

A separate issue about the logic of mathematics is illustrated by the following paragraph:

> We can represent the complex number $z = x + iy$ by the point (x, y) in the complex plane, where the rectangular Cartesian coordinates x and y have their usual meanings. Alternatively, we may represent $z = x + iy$ by a directed line segment (vector) from the origin to the point (x, y). … It is also convenient to express a complex number z in polar coordinates $(r, ø)$, where r is the distance of z from the origin, and $ø$ is the angle which the line from the origin to z makes with the positive real axis.

There are four options here for the diagrammatic representation of a complex number: point or vector in Cartesian coordinates, or point or vector in polar coordinates. This is *arbitrary* as indicated by "can", "may", "alternatively". We can use one or the other depending on what is useful. At earlier levels, mathematics is usually presented as predetermined. At advanced levels the arbitrary and contingent nature of mathematics emerges. We can define objects as we want them to be. There are subtle changes of language indicating this shift. For example, the phrase "we can…" can mean "it is possible or not…" or it can mean "it is the case". Consider the difference between "I can go to the movies (I am permitted if I so choose)" and "I can drive a car" (this is a skill I have).

Phase 2: Testing a Sample of EAL Students

The 12 Chinese-speaking students who agreed to provide complete data divided into two groups on their DELNA test of English proficiency.

Table 14 **English proficiency results**

Averages	**Years learning maths in English**	**DELNA listening**	**DELNA reading**	**DELNA writing**	**DELNA average**
Lower proficiency English group	10.7	4.5	4.5	5.3	4.8
Better proficiency English group	4.2	5.9	5.7	6.7	6.1
DELNA is a 9 -point scale. English language support strongly recommended for 6 or lower. No-one in lower proficiency group outperformed anyone in better group. (Average score of the two groups is significantly different at 1 percent level: p = 0.001.)					

It was decided to analyse the other results by comparing these two groups.

The students were asked whether they understood various aspects of the language used in their mathematics course.

Table 15 **Self-reported understanding**

	Lectures	**Notes & texts**	**Assts. & tests**	**Other students**	**Vocab.**	***Grammar***	**Instructns.**	**Logic of maths**
Lower profic. English group	212131 1.7	212141 1.8	222122 1.8	111122 1.3	312222 2.0	212311 1.7	312212 1.8	211422 2.0
Better profic. English group	121111 1.2	121111 1.2	111111 1.0	211111 1.2	322112 1.8	222211 1.7	111111 1.0	311111 1.3
Likert Scale: **1 = No difficulty 2 = A little difficult 3 = Some difficulty 4 = A lot of difficulty**								

(The top group of numbers are individual scores, the number underneath is the average.)

The first question in the test was a repeat of a question that had been done poorly by all students in a previous class test. It was given again, but this time with step-by-step written instructions on how to do the question. The following results were obtained.

Table 16 **Change in marks when a test question is explained**

	Individual change in mark (maximum mark was 14)	Average change in mark for the group
Lower proficiency group	–4, –4, –2, +1, +1, +2	–1.0
Better proficiency group	–2, +2, +3, +3, +4, +6	+2.7

(Average scores are significantly different at 5 percent level, p = 0.039.)

These results indicate that the lower proficiency English group become more confused when explanations are given in English.

Density of multiple adjectives was tested by a question in which a *K-function* was defined as a non-negative, continuous, everywhere decreasing, function. (These should be familiar terms at this level.) Twelve graphs were presented, and students were asked to identify the K-functions. Two of the 12 graphs did illustrate such functions, the other 10 graphs were not K-functions for one of the four possible reasons.

In the lower proficiency English group two of the six students missed one of the two K-functions, but they also incorrectly identified 13 other graphs, an average of more than two incorrect responses each. In the better proficiency English group two out of the seven students also missed one of the two K-functions, and these two students also wrongly identified one other graph. All other students were completely correct. This is indicative evidence that the lower proficiency group was less able to hold four characteristics in mind at the same time.

The syntax of sentence structure was tested by a question in which there were four statements, two of which were definitions and two of which were theorems. The essential difference was the verb: "is" being used for definitions ("A is B"); and "has" being used for theorems ("A has B"). This question was performed 100 percent correctly by the better proficiency English group. The lower proficiency English group, however, performed no better than chance. Evidence that they were guessing was enhanced by the fact that all six students identified the two theorems as one theorem and one definition. That is, they did not even identify that they were the same as each other.

Logical complexity was tested in two questions. The first question involved a mathematical statement of the form "A is B if there is a C such that D is E". Four derivative statements were given, two of which were correct, and two of which did not follow from the original. Students were asked to say whether the statements were true or false. Neither the lower proficiency nor the better proficiency group answered this question better than chance.

The second question gave a mathematical theorem and then asked students to:
(a) write it in the form "If A then B";

(b) write it in the form “If B then A” (and say whether it is true or not);
(c) write it in the form “If not-B then not-A” (and say whether it is true or not).

In the lower proficiency English group only one of the six students was able to do this, the others were completely unable to do this task. In the better proficiency group three of the seven students completed it correctly, one managed (a) and (b), two managed only (a), and the other could not do it.

Of all the features tested, the logical complexity questions appeared to cause problems for both groups of students, but were apparently completely beyond the lower proficiency group.

Phase 3: Confirming Results on a Larger Group

The test given to the two larger classes in the second semester was identical except that it did not have the repeat question from an earlier class test in an explanatory form. The results are given in Table 4 below (it includes the data from Phase 2). The problems experienced by EAL students in the first test were repeated with the larger group, and thus the conclusions are supported. The L1 students returned almost perfect answers, indicating that they have no difficulty with mathematical discourse of this kind.

One further piece of analysis was undertaken. Conventional wisdom in the Department of Mathematics was that EAL students did less well in higher-level courses. Thus grades were investigated for students over their undergraduate studies. It was found that all L1 students maintained the same grade average each year. EAL students, however, experience some drop. The students in Phase 2 of the study, for example, experienced an average drop of 1.3 grades. We do not regard this data as reliable because there are insufficient students with complete data and the variability between students is very large.

Table 17 **Data for large classes and L1 students**

<table>
<tr><th></th><th colspan="6">EAL students
(Chinese = 33; PI = 5; Indian = 7; Other = 8)</th><th>English L1 students
(n = 8)</th></tr>
<tr><td rowspan="2">Self-reported understanding of the course
1=No difficulty
2=Little diff.
3=Some diff.
4=Lot diff.</td><td>Percent response on Likert Scale</td><td>1</td><td>2</td><td>3</td><td>4</td><td>Av. Rating</td><td rowspan="2">Three students reported a little difficulty with the (Indian-background) lecturer.
No other difficulty reported.</td></tr>
<tr><td>Chinese speakers
Pacific lang. spkrs
Indian lang. spkrs.
Others</td><td>36
0
40
55</td><td>45
35
29
27</td><td>16
58
15
14</td><td>3
8
17
5</td><td>1.9
2.7
2.1
1.7</td></tr>
<tr><td>Multiple adjectives:
The K-function</td><td colspan="6">Chinese speakers: 7/33 missed one of the correct graphs. On average each student selected one other graph wrongly.
All others combined: 8/20 missed one or more correct graph. On average each student selected 2.4 other graphs wrongly.</td><td>All students identified the two correct graphs. Three students wrongly identified one other.</td></tr>
<tr><td>Syntax:
Sentence structure</td><td colspan="6">Chinese speakers: 84% correct responses.
All others combined: 71% correct responses.</td><td>All students answered these questions 100% correctly.</td></tr>
<tr><td>Logic:
Question 1</td><td colspan="6">Chinese speakers: 50% correct, i.e. exactly the same as chance.
All others combined: 54% correct, also no different from chance.</td><td>One student got one of the four items wrong. All others correct.</td></tr>
<tr><td>Logic:
Question 2</td><td colspan="6">Chinese speakers: 11/33 students completely correct. The other 22/33 were 26% correct. Seven students tried to write it symbolically.
All others combined: 4/20 students completely correct. The other 16/20 were 33% correct. Four students tried to write it symbolically.</td><td>One student got one of the three items wrong. All others correct.</td></tr>
</table>

Discussion

The first unexpected result from this study was the number of third-year mathematics students who do not have what is generally regarded as the minimum levels of English language proficiency to undertake university study. Half of the sample in Phase 2 were significantly below an equivalent IELTS Band of 6. These all appeared to be long-term New Zealand residents who had studied mathematics in New Zealand secondary schools. International students must pass an IELTS requirement, so they have better proficiency. The new literacy requirements for university entrance may change this situation in the future.

This study confirms that EAL students suffer a disadvantage due to language when studying mathematics. The evidence presented here is that the language requirements at third-year level are much greater (and are new) compared with those in first-year. It is concluded, therefore, that the levels of disadvantage are greater than those measured in first-year students, that is, as great as in other subjects. While the better proficiency group does not have such disadvantage on vocabulary and syntax items, the logical complexity of third-year mathematics in English is well beyond the capabilities of many of them. The extra time spent in a New Zealand tertiary institution does not seem to improve their English proficiency sufficiently to make up for these linguistic demands. The L1 students in this study did not have any language problems.

The EAL students are unaware of their difficulties. Apart from the Pacific Island students, they report only a little difficulty with any language aspect of the course, when the evidence is that they are in serious trouble. The Pacific Island students probably have better oral fluency (anecdotal evidence, this was not tested in the study)—perhaps this makes them aware of the textual and listening problems with mathematical discourse? Another explanation is possibly that EAL students from Asian countries have experienced success in mathematics at earlier levels due to their better mathematical backgrounds, and believe that therefore they must understand the work. In fact, they have developed successful strategies that work with the computational examples and exercises that predominate at first-year level, but are not aware that the nature of the discourse has changed.

There are some limitations in this study. It has not been possible to moderate the results for mathematical ability. The lower proficiency English group has poorer grades in previous courses, and did less well in their examinations at third-year level. However it is reasonable to assume that some, possibly a large amount, of this is due to language difficulties which is what we are trying to measure. None of the mathematics presented in the test was beyond second-year, and all students had passed second-year courses. The L1 students, in particular, were all very good students, so direct comparisons are dangerous. However, their ability with the test was near perfect, and better than EAL students with similar grades. Not one of the five EAL A+ students scored perfectly on this test of language, and the one English student with less than A grades (best grade B) did score perfectly.

Finally, we believe that the linguistic difficulty is even more subtle than this study indicates. One of the researchers found that the format of an examination question caused problems only for EAL students. A set was referred to in the opening sentence of the question and should have been used in both parts (a) and (b). EAL students used it in part (a) but ignored it in part (b). All L1 students understood what was required.

Summary

Observations of third-year university mathematics classes revealed that the relationship between language and mathematics is much more complex at this level than earlier levels, and presents new linguistic challenges. Testing EAL students in these classes reveals particular difficulties with multiple indicative adjectival phrases, the syntax of mathematical discourse, and, especially, with logical complexity. Those students with English proficiency at levels regarded suitable for university study only had difficulty with logical complexity. There were a few students with strong mathematics grades who were minimally affected by English language issues.

We conclude that learning mathematics in English is as difficult for EAL students as learning any other subject.

Recommendations

- All undergraduate mathematics students should be tested for their English proficiency.
- All mathematics lecturers need some professional development in how to help EAL students understand the linguistic necessities of the subject.
- Specific support courses for English in mathematical discourse need to be established for EAL students in the mathematical sciences. These may include opportunities for using their first language provided that the focus of assistance is on the bridge between this language and mathematics in English.

References

Adler, J. (1998). A language of teaching dilemmas: Unlocking the complex multilingual secondary mathematics classroom. *For the Learning of Mathematics, 18,* 24-33.

Barton, B., & Neville-Barton, P. (2003). Language issues in undergraduate mathematics: A report of two studies. *New Zealand Journal of Mathematics, Vol 32*, Supplementary Issue, 19-28.

Barton, B., & Neville-Barton, P. (2004). *Undergraduate mathematics learning in English by speakers of other languages.* Paper presented to Topic Study Group 25 at the 10th International Congress on Mathematics Education, Copenhagen, July, 2004.

Dale, T., & Cuevas, G. (1987). Integrating language and mathematics learning. In J. Crandall (Ed.), *ESL through content-area instruction* (pp. 9-52). Englewood Cliffs, NJ: Prentice Hall Regents.

Halliday, M. (1978). *Language as social semiotic*, London: Edward Arnold.