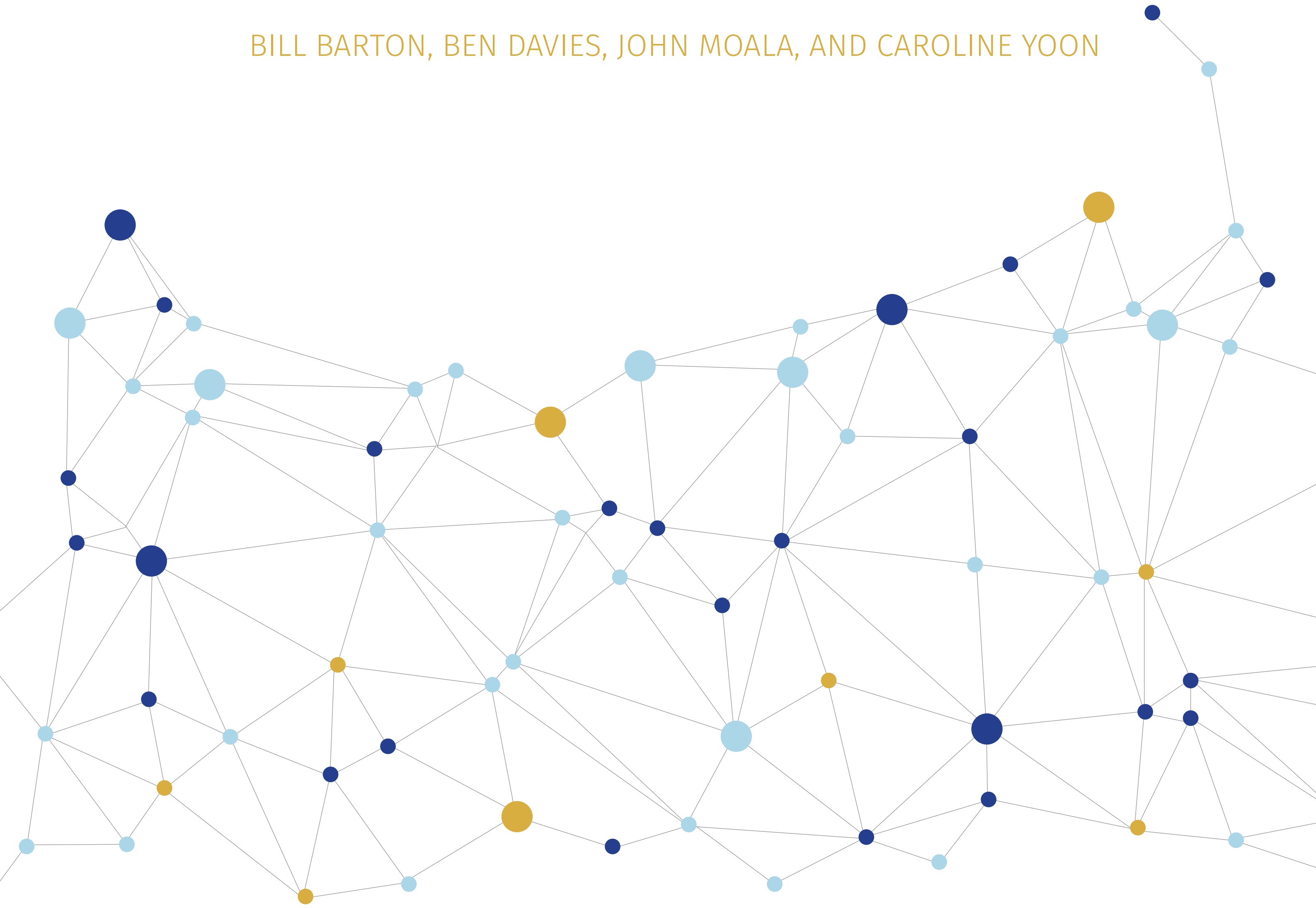
GUIDE #6

LEARNING IN UNDERGRADUATE MATHEMATICS:
THE OUTCOME SPECTRUM (LUMOS)
"HOW TO" GUIDES

Generate conceptual readiness

BILL BARTON, BEN DAVIES, JOHN MOALA, AND CAROLINE YOON



TEACHING & LEARNING RESEARCH INITIATIVE
NĀU I WHATU TE KĀKAHU, HE TĀNIKO TAKU

AKO AOTEAROA
NATIONAL CENTRE FOR TERTIARY TEACHING EXCELLENCE

NPF NATIONAL PROJECT FUND
AKO AOTEAROA

THE UNIVERSITY OF AUCKLAND
Te Whare Wānanga o Tāmaki Makaurau
NEW ZEALAND

A series of "How to" guides

“HOW TO” GUIDE #6: This guide is one of seven produced by the project Learning in Undergraduate Mathematics: The Outcome Spectrum (LUMOS). LUMOS examined the learning outcomes of undergraduates in the mathematical sciences.

The full list of titles in the series is:

"How to" Guide #1: Implement team-based learning

"How to" Guide #2: Implement semi-authentic mathematical experiences

"How to" Guide #3: Shift responsibility for learning onto students

"How to" Guide #4: Monitor feelings and beliefs about the mathematical sciences

"How to" Guide #5: Monitor the development of mathematical communication

"How to" Guide #6: Generate conceptual readiness

"How to" Guide #7: Develop mathematical habits

LEARNING IN UNDERGRADUATE MATHEMATICS: THE OUTCOME SPECTRUM (LUMOS).

“HOW TO” GUIDE #6: GENERATE CONCEPTUAL READINESS

Authors
Bill Barton, Ben Davies, John Moala, Caroline Yoon

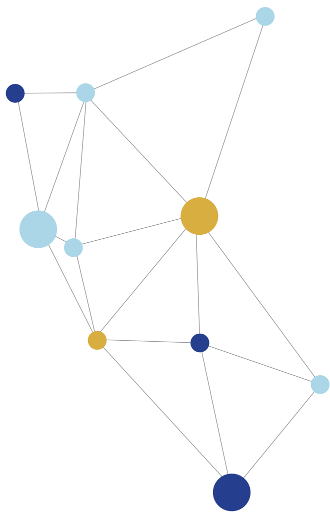
This work was supported through the Teaching and Learning Research Initiative (TLRI) and Ako Aotearoa National Project Fund 2012. A series of “How to” guides and the research report can be downloaded at www.tlri.org.nz/undergraduatemathematics or akoaotearoa.ac.nz/undergraduate-mathematics. The guides are also available for purchase at www.shop.akoaotearoa.ac.nz.

Publisher
Ako Aotearoa – The National Centre for Tertiary Teaching Excellence
PO Box 756
Wellington 6140

Published
August 2017

ISBN 978-0-947516-57-4 (online)
ISBN 978-0-947516-58-1 (print)

Design by Two Sparrows, www.twosparrows.co.nz



This work is licensed under the *Creative Commons Attribution-NonCommercial-ShareAlike* 4.0 International License. To view a copy of this license, visit http://creativecommons.org/licenses/by-nc-sa/4.0/ or send a letter to Creative Commons, PO Box 1866, Mountain View, CA 94042, USA.

Contents

Conceptual Readiness

Conceptual readiness is the idea that, in order to get the most out of a lecture, students need to be conceptually prepared for the information they are about to receive.

We believe that the traditional lecture format can be a powerful method of delivery, provided that all students are conceptually ready for what is being presented. Currently lectures proceed with many students not understanding the need for, or the concepts behind, the topic. Prior to a lecture, students need opportunities to struggle, to notice the limits of their own knowledge, and to become aware of what they do not know. Ideally students need to be mentally at the edge of a deep mathematical idea. This is what is meant by "conceptually ready".

Our experience is that it is possible, with suitably designed tasks, to prepare students to better receive lecture material. The design and delivery of those tasks is described in this guide.

Conceptual learning has become a focal point of literature on mathematics education since Richard Skemp's writing on relational and instrumental understanding (Skemp, 1976). The former is characterised as "knowing what to do and why", the latter as "rules without reasons". It is an unfortunate side effect of the focus on assessment and grades that students are able to pass many undergraduate courses with very little relational understanding, but using their instrumental understanding.

Contemporary educators are much more nuanced in their descriptions of types of understanding, and recognise overlaps and the dependence on context. Furthermore, the learning value of different kinds of understanding are now better understood. Nevertheless, undergraduate lecturers still strive to help students become deep learners. Not only do they want students to know what to do, and why, but they also must be able to anticipate the outcome of their work, have a wider vision of the topic, know alternative methods of approaching a problem, and be able to link their learning to other things they know (see, for example, Keene & Fortune, 2016).

In secondary education, work has been done on Mathematics Eliciting Activities (MEAs). This is an approach used to develop realistic scenarios, which would enforce mathematical thinking in order to reach a resolution (Lesh, Hoover, Hole, Kelly, & Post, 2000). MEAs have been used in New Zealand; see Yoon, Patel, Radonich, & Sullivan (2011).

However, the principles of designing MEAs need to be adapted for undergraduate level because not all important concepts are easily approached through a realistic scenario—for example, mathematical induction.

A research group associated with the LUMOS project have undertaken this adaptation and trialled some tasks designed for conceptual readiness. Much of the work focused on graph theory and, in particular, proof by induction.

Principles for Designing Readiness Tasks

The activities outlined in this booklet are designed to be pre-lecture activities undertaken in a tutorial-type setting. Although it might be possible to set tasks as homework, our experience is that better thinking takes place in groups.

The aim is to create learning environments where students will struggle and get stuck (but not give up), and thereby force deeper mathematical thinking. Such thinking will prepare them for the mathematics to come in their courses.

In most situations, where students are stuck it is because they cannot access the material they need to know, either because it has been forgotten, the appropriate material was not identified, or perhaps it was never learned in the first place. Therefore, a useful approach is to create a situation that does not rely on some piece of learned material, but rather requires students to think deeply about the situation, bringing a variety of ideas and approaches.

We have developed a set of principles for designing tasks that will achieve this goal. Four principles describe the way the student accesses the activity. The other principles describe the work to be done.

Access principles

- Interest principle: The task must be interesting for most students.
- Rule play principle: The task must be amenable to rational analysis.
- Messy play principle: The task must provide the opportunity for exploration and creativity.
- Alignment principle: The task must align with the target concept.

Work principles

- Product principle: The task must result in some mathematics being produced.
- Necessity principle: Students must be able to see the need for the product.
- Focus principle: The task must focus on bringing some mathematical topic to the fore.
- Accountability principle: Each student must be accountable to someone for their reasoning and thinking on the task.
- Elegance principle: The result of the task must have aesthetic value.
- Generalisation principle: The result of the task must be easily adaptable for other situations.

Sample Approaches

Getting stuck with Combinatorics

The first sample approach describes an attempt to induce “stuckness” in a group of students working with combinatorics. It illustrates how a sequence of tasks can be designed to lead students into readiness for complex mathematical ideas.

A group of third year undergraduate students volunteered for a trial in which they worked on a sequence of four tasks in combinatorics. The tasks were designed to produce conceptual thinking by creating situations in which they would get stuck but would continue to work.

The required mathematics for the tasks had already been covered in their courses (and they had access to a basic text), but the tasks were not amenable to a direct approach using this material in familiar ways. In addition, these students had been in a Team-Based Learning environment (see Guide #1 in this series). Therefore, they were experienced with team work, (and hence the Accountability Principle was already achieved), and enjoyed this mode of learning. Finally, these students were well-motivated, and so already showed an interest in the topic.

The first task begins with the story of the well-known Erdös-Bacon number, introducing the notions of a network and degrees of connectedness. The problem statement poses a situation in which a New Zealand tourist acquired a new set of Kiwisms. Upon returning home, she wishes to spread one of the new words throughout her network of friends and must devise an algorithm that would determine the first person with whom she must share the word, so as to reach everyone in the network in the least amount of time. The Interest Principle is thus achieved. From a graph-theoretical point of view, the task is an introduction to notions of connectivity, such as vertex-degree, distance, eccentricity, centrality, and connectedness. The Rule Play Principle therefore applies.

The second task revolves around a given algorithm that is deliberately defective in several aspects. Students are provided with a number of scenarios relating to the algorithm. They are required to identify issues with it (Messy Play Principle) and to propose improvements. As a result, they are encouraged to refine their own algorithm devised in the first task (Alignment and Product Principles).

In the third task the concept of a cut vertex is introduced under the name “pivotal person”. The original context (Task 1) is altered by removing the assumption that any person who learns the new word shares it with all his/her connections. Students are required to test (and modify) their initial interpretation of pivotal person, think of ways that a pivotal person can be identified in an arbitrary network, and begin to think about ways of extending and generalising the concept towards (say) some notion of pivotal people. The third task brings the Necessity, Focus and Generalisation Principles into play.

Given the aim to bring about stuckness, the final task is centred on the concept of separating sets, which is to some extent an extension of the pivotal person introduced in the third task. The primary goal is to observe how students go about re-structuring what they have learned/seen (in previous tasks) in order to make sense of a novel situation.

For full details of the task sequence, please see Appendix A of Moala (2015).

The results of the study included:

- The sorts of things that were not helpful to the group when they were stuck (e.g. an overemphasis on dominant, but often inadequate ideas).
- General unfavourable characteristics of the group's thinking that were perhaps induced by the condition of being stuck (e.g. increased immersion in instant gratification).
- The sorts of things that were helpful to the group when they were stuck (e.g. re-presenting the problem in a different manner).
- Some things that might aid a novice's ability to cope with being stuck (e.g. developing the ability to sensitise oneself to being stuck).

This preliminary study had several limitations (described in the work), but showed how significant work (the equivalent of four tutorial sessions) led to a group of students deeply engaging mathematically with a situation beyond their individual levels of expertise. This was done in such a way that their further study of combinatorics was enhanced—their readiness for formal presentation of the concepts they "playfully" approached was significantly enhanced.

Quasi-induction Tasks

The second sample approach was the trial that led to the formation of several of the principles mentioned above. The trial involved developing tasks that would lead students towards the idea of mathematical induction. Earlier literature had identified early concepts leading to induction, labelled quasi-induction. Two tasks were developed aimed at these concepts, and trialled with three groups of first year mathematics major undergraduates.

The first task involved setting up dominoes on end so that, with one touch, a whole row would fall over. By creating branching rows, questions were asked like: "make a row of dominoes such that it is possible to make 1, 2, 3, 4, 6, 7, 8, or 9 dominoes fall. Note, it must not be possible to make 5 dominoes fall".

The second task involved cutting a disc ("cake") into as many pieces as possible with 1, 2, 3, 4, ... straight cuts.

Both tasks involved a considerable number of subsidiary questions and requests for justifications. See the Appendices in Davies (2011) for details.

When trialled, the first task did produce some (but not all) proof schemes that were related to quasi-induction. However, the second task produced mostly empirical proof schemes as students resorted to "find a formula". Analysis of the student work on the tasks exposed deficiencies in the tasks. The analysis led to the formation of the set of principles above.

Using Counter-Examples and other resources

Schwartz and Martin (2004) also focused on preparation for future learning, and advocated "invention activities" as a way of doing this. Their work was with junior secondary students in a statistics context, but raised the idea of creative and explorative activities to generate conceptual readiness.

Others working in undergraduate mathematics have trialled tasks for use by undergraduate students to promote mathematical thinking. Breen and O'Shea (2011) report on tasks that are used as homework problems.

Other writers have proposed different activities directly aimed at deepening conceptual understanding. In New Zealand, Sergiy Klymchuk at Auckland University of Technology investigated the use of counter-examples in another Ako Aotearoa project (Klymchuk, 2009).

The use of writing to enhance conceptual understanding has long been promoted as a useful technique, although few practical approaches have emerged. Reynolds, Thaiss, Katkin, and Thompson (2012) review the literature and recommend assignments that 1) focus critical reflection on one's beliefs regarding knowledge, and 2) assignments that engage the student in formulating a reasoned argument.

Concluding Comments

We strongly recommend using at least some tutorial time to prepare for the ideas being introduced in lectures, as opposed to practising or revising ideas that have already been presented. This is the basic idea of conceptual readiness.

Exactly how this preparation takes place is deeply context dependent. It is particularly affected by the content of the course, and the background and habits of the students. This guide contains some examples of tasks that were tried for this purpose.

At this point we recommend:

- Tasks should be attempted in groups, probably of three students or possibly pairs of students.
- Tasks should be designed with the ten principles mentioned on page 5 in mind.
- The tutorials should be held immediately prior to the lecture in which the main concept is introduced.
- Work on these tasks should not become part of the course assessment.

We would very much like to hear of other attempts, successful or not, so that we may learn as a community. Please contact the Department of Mathematics at the University of Auckland (email: enquiries@math.auckland.ac.nz).

References

Breen, S. & O'Shea, A. (2011). The use of mathematical tasks to develop mathematical thinking skills in undergraduate calculus courses – a pilot study. In Smith, C. (Ed.), ***Proceedings of the British Society for Research into Learning Mathematics*** 31(1). March 2011. Available from Informal Proceedings 31-1 (BSRLM) at bsrlm.org.uk, 43-48.

Davies, B. (2015). ***New Principles for Designing Quasi-Induction Tasks***. Unpublished BSc Honours Dissertation. The University of Auckland

Keene, K. & Fortune, F. (2016). A Framework for Integrating Conceptual and Procedural Understanding in the First Two Years of Undergraduate Mathematics. ***On Teaching and Learning Mathematics, AMS Blog***, 16 May, 2016. Accessed 05.09.16 at http://blogs.ams.org/matheducation/#sthash.77iyjtqp.dpuf

Klymchuk, S. (2009). Using counter-examples to enhance learners' understanding of undergraduate mathematics. In Good Practice Publication Grants (Online) (9 pages). New Zealand: Ako Aotearoa National Centre for Tertiary Teaching Excellence. Retrieved from http://akoaotearoa.ac.nz/download/ng/file/group-3300/using-counter-examples-to-enhance-learners-understanding-of-undergraduate-mathematics.pdf

Lesh, R., Hoover, M., Hole, B., Kelly, A., & Post, T. (2000). Principles for developing thought-revealing activities for students and teachers. In A. E. Kelly & R. A. Lesh (Eds.), ***Handbook of research design in mathematics and science education*** (pp. 591–646). Mahwah, NJ: Lawrence Erlbaum Associates.

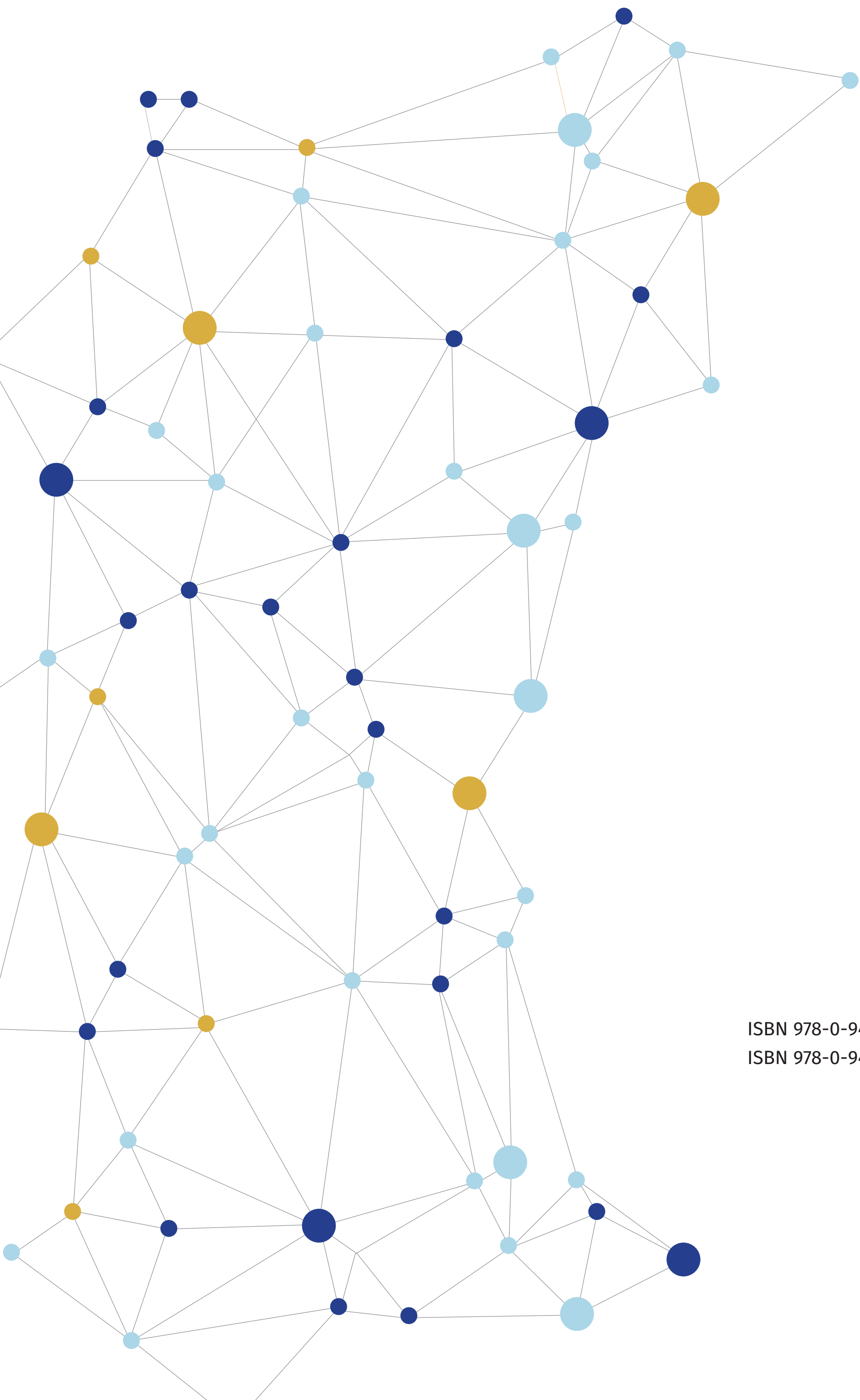
Moala, J. (2015). ***On Being Stuck: A preliminary investigation into the essence of mathematical problem solving***. Unpublished MSc thesis. The University of Auckland.

Reynolds, J., Thaiss, C., Katkin, W., & Thompson, R. (2012). Writing-to-learn in undergraduate science education: a community-based, conceptually driven approach. ***CBE Life Sciences Education***, 11(1), 17-25. DOI 10.1187/cbe.11-08-0064.

Schwartz, D. L. and Martin, T. (2004). Inventing to prepare for future learning: The hidden efficiency of encouraging original student production in statistics instruction. ***Cognition and Instruction***, 22, 129–184.

Skemp, R. (1976). Relational and instrumental understanding. ***Mathematics Teaching***, 77, 20-26.

Yoon, C., Patel, A., Radonich, P., & Sullivan, N. (2011). ***LEMMA: Learning Environments with Mathematical Modelling Activities. Final Report.*** Wellington: Teaching & Learning Research Initiative.



ISBN 978-0-947516-57-4 (online)
ISBN 978-0-947516-58-1 (print)